Competitive Entry and Endogenous Risk in the Foreign Exchange Market

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Recent evidence shows that higher trader participation increases exchange rate volatility. To explore this linkage, we develop a dynamic model of endogenous entry of traders subject to heterogenous expectational errors. Entry of a marginal trader into the market has two effects: it increases the capacity of the market to absorb exogenous supply risk, and at the same time it adds noise and endogenous trading risk. The competitive entry equilibrium is characterized by excessive market entry and excessively volatile prices. A positive tax on entrants can decrease trader participation and volatility while increasing market efficiency.

Recent empirical research on the microstructure of the foreign exchange market has documented increased exchange rate volatility for periods of higher trader participation. Ito, Lyons, and Melvin (1998) find that lunch-hour exchange rate variance doubles in 1994 when Tokyo traders were permitted to participate in the market making between 12:00 P.M. and 1:30 P.M. The evidence poses a wide range of questions: Why does greater market participation increase volatility? Do competitive financial markets like the foreign exchange market provide the right incentives for market entry of speculative traders? And if not, can a

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Tobin tax on financial institution induce their exit, reduce speculative trading, and stabilize the market price?[1]

This article provides a new perspective on these questions. We model endogenous competitive entry into a dynamic speculative market. Financial institutions make fully rational hiring decisions about whether or not to hire a trader who conducts the trading on behalf of the institution. Traders seek profitable trading opportunities based on inference of current and past exchange rates. Their trading is subject to temporary expectational errors about the excess return relative to the optimal prediction of the dynamic exchange rate filtration.[2] Unlike in much of the literature on so-called noisy rational expectation models, noise results from the expectational errors of the traders and its level is endogenously determined by the entry decisions of financial institutions.

Central to the analysis is the dual effect of trader entry on the risk-sharing capacity of the market and on the information content of the equilibrium price. A marginal trader in our model enhances risk sharing only at the cost of creating more endogenous noise with a negative externality on the inference abilities of all incumbent traders. Depending on the relative importance of both effects, marginal entry may increase or decrease price volatility. The competitive market entry decision of a financial institution fails to internalize both externalities. We show that for a high information content of the market price, the negative information externality tends to dominate the risk-sharing benefit. Competitive entry of financial institutions can produce excessive market entry with excessively volatile exchange rates.[3] Exogenous legal constraints on market participation like the lunch-hour rule in the Tokyo market can decrease volatility. The allocational efficiency of the competitive entry equilibrium can also be improved by a positive tax rate on financial institutions, reducing both market entry and volatility.

The previous literature [Hirshleifer (1988), Pagano (1989), and Allen and Gale (1994)] has pointed out that endogenous entry decisions may give rise to multiple equilibria. Limited trader participation is generally associated with an inefficient high-volatility equilibrium, while full trader participation gives rise to a more efficient low-volatility equilibrium. Entry in these settings induces a positive risk-sharing externality. By contrast, endogenous noise in our model can explain multiple equilibria characterized by a positive correlation between trader participation and volatility. Our result is

[1] For a recent debate of taxation as a mean to reduce exchange rate volatility, see Haq, Kaul, and Grunberg (1996).

[2] It is assumed that the unconditional mean of the expectational error is zero. The expectational error is therefore not systematic as assumed by De Long, Shleifer, Summers, et al. (1991). For evidence on expectational errors see Frankel and Froot (1989, 1990, 1993).

[3] Campbell and Clarida (1987) find, for example, that movements in the expected interest rate differential have not been large enough or persistent enough to account for the variability in the real dollar exchange rate.

easier to reconcile with the stylized fact of a positive correlation between the number of transactions and volatility in stock markets [Jones, Kaul, and Lipson (1994)]. Negative information externalities due to entry were first highlighted by Stein (1987). But Stein uses a static model with exogenous entry decisions. Our analysis underlies a dynamic market model and we explicitly characterize the size of the trader set in equilibrium. Endogenous entry decisions also differentiate our work from exogenous participation restrictions assumed by Merton (1987), Basak and Cuoco (1997), and Shapiro (1998) in a multiasset model.

Any model of endogenous entry has to address the survivorship issue with respect to traders of inferior trading abilities [Friedman (1953)]. De Long, Shleifer, Summers, et al. (1991) show that irrational traders might obtain higher returns at higher risk than fully rational traders and argue that irrational traders might survive. Palomino (1996) points out that survivorship in a competitive market requires risk illusions on the part of investors. Such risk illusions are less plausible for the foreign exchange market where traders are predominantly hired by sophisticated financial institutions. Our model approach differs in that we assume a single trader type with heterogenous expectational errors of identical variance. This avoids the typical dichotomy between fully rational and irrational traders and the associated survivorship objection.

Three empirical implications about volume, volatility, and trading profits can be highlighted. Explaining volume remains a challenge for microstructure models.[4] Heterogenous expectational errors in our central market framework imply a relatively high intramarket trading volume between traders. Our benchmark model predicts that approximately 41% of the trading volume is intramarket trading. However, this falls short of the 70% observed in the foreign exchange market.[5] Accounting for a decentralized market structure appears essential for explaining the observed intramarket volume.[6] Second, our model can explain exchange rate heteroscedasticity. The competitive market entry process allows for two stable equilibria; one with a small trader set and low volatility and a second one with a large trader set and high volatility. Third, we derive the model implications for trading profits. The latter are predicted to increase in volatility even if entry of traders is endogenous. We test for a positive correlation between profits and volatility using data on the trading profits of 20 large U.S. banks and find evidence for a correlation between trading profit and volatility.

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[4] For a criticism of volume implications of asymmetric information models, see Harris and Raviv (1993) and Kandel and Pearson (1995).

[5] According to the Bank for International Settlements, trading on behalf of customers in April 1995 amounted to 25.5% in London, 26.5% in Tokyo, and 44% in New York. The three markets together imply an intramarket trading share of 69.6%.

[6] See Rerraudin and Vitale (1996) for a model of a decentralized market.

The remainder of the article is organized as follows. Section 1 presents the model. The dynamic exchange rate equilibrium is derived in Section 2 for an exogenous number of financial institutions. Section 3 solves the market entry problem of the financial institutions and we discuss the efficiency properties of the competitive entry equilibrium. The empirical implications for the trading volume and trading profits are explored in Section 4. Section 5 concludes.

1. The Model

Consider the foreign exchange market in which foreign currency is continuously traded over an infinite horizon.[7] All insurance and consumption smoothing motives of trading are assumed to be absent. The financial market exists to balance random fluctuations in the trade balance between two countries.

Assumption 1 (Net Supply). *The net supply of foreign currency $Q(\Theta_t, P_t)$ at time $t \in [0, \infty)$ is linear in the deviation of the exchange rate P_t from its long-run equilibrium $\overline{P}$. The net supply is shifted by a stochastic process Θ_t. For a constant parameter $\gamma \geq 0$, we assume*

$$Q(\Theta_t, P_t) = \Theta_t + \gamma \left(P_t - \overline{P}\right).$$

A depreciation in the home country (higher exchange rate P_t) improves its trade balance and therefore increases the supply of foreign exchange in the home country. For the special case where $\gamma = 0$, the supply of foreign exchange is completely price inelastic, an assumption usually made in noisy rational expectations models. The random fluctuations in the net supply create profitable intertemporal trading opportunities, which can be exploited through speculative trading.

Our framework distinguishes between financial institutions, who make hiring decisions, and traders, who undertake the trading on behalf of the financial institutions. Traders have imperfect trading abilities and their demand can deviate from the optimal currency demand. Financial institutions on the other hand make optimal hiring decisions based on rational expectations about the traders' imperfect trading abilities. We denote the (countable, infinite) set of financial institutions by $\mathcal{I}$.[8] At time $t = 0$, each financial institution $i \in \mathcal{I}$ faces the decision of hiring a single trader ($y^i = 1$) or not ($y^i = 0$). A subset $\mathcal{T} \subseteq \mathcal{I}$ of financial institutions enters the market by

[7] Our framework is not specific to the foreign exchange market, but can similarly be applied to an equity market with a constant dividend flow. Yet the framework ignores private information considered important for equity markets.

[8] An infinite number of financial institutions simplifies the aggregation problem and represents the limit case of a "large" competitive market.

hiring a trader, whose asset demand X_t^i generates a stochastic cumulative net trading profit Π_t^i. We assume that operating in the market is costly. If a financial institution enters ($y^i = 1$), its net profit flow $d\Pi_t^i$ in a time interval dt consists of the flow of (negative) operating costs $-c(\cdot)dt$ and the gross trading profit $X_t^i dR_t$, where dR_t denotes the excess return on a foreign currency position. The operating costs $c(\cdot)$ are deterministic and may increase in the number of entrants as financial institutions compete for scare resources. The excess return dR_t on a unit of foreign currency consists of the capital gain dP_t and the cost of capital $-\overline{r}P_t dt$ for a nominal interest rate differential $\overline{r} \equiv r - r^*$ between the domestic and foreign riskless rate.[9] Financial institutions which do not enter the market ($y^i = 0$) make zero profits.

Assumption 2 (Financial Institutions). *The optimal entry decision $y^i \in \{0, 1\}$ of any financial institution $i \in \mathcal{I}$ at time $t = 0$ maximizes the unconditional expectation of the objective function J^i,*

$$\max_{y^i \in \{0,1\}} E\left[J^i(0)\right], \quad J^i(t) \equiv \int_{s=t}^{\infty} e^{-r(s-t)} \left[d\Pi_s^i - \tfrac{1}{2}\rho\left(d\Pi_s^i\right)^2\right]$$

$$s.t. \quad d\Pi_t^i \equiv \begin{cases} -c(.)dt + X_t^i dR_t & \textit{for} \quad y^i = 1 \\ 0 & \textit{for} \quad y^i = 0 \end{cases}$$
$$dR_t \equiv dP_t - \overline{r}P_t dt.$$

The intertemporal objective function $J^i(t)$ for the financial institution is defined as the expected present value of a quadratic utility function $d\Pi_t^i - \frac{1}{2}\rho(d\Pi_t^i)^2$ in the profit flow $d\Pi_t^i$.[10] The parameter ρ measures the institution's risk aversion. Financial institutions make utility maximizing market entry decisions. They have rational expectations about the stochastic profit flow $d\Pi_t^i$ that results from a hiring decision. If the expected profit flow is too low relative to the level of trading risk, a financial institution decides not to enter the market ($y^i = 0$).

Next, we specify the currency demand of the traders. The literature has often distinguished two extreme trader types—fully rational traders and fully irrational noise traders. In order to allow for an analysis of the entry problem, we merge these two types into a single trader type with a rational and an irrational demand component. Traders in our model are therefore neither fully rational (because of the expectational error component in their demand) nor fully irrational (because of the rational demand component

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[9] In the case of an equity market, the term $\overline{r} = r - d$ denotes the difference between the riskless rate r and the dividend payment d.

[10] The quadratic utility framework simplifies the analysis relative to the common CARA formulation for a dynamic model. It allows us to ignore the cross correlations of the excess return with the state variables in the trader's asset demand [see Wang (1993)].

reflecting information in prices). Formally, let $(\Omega, \mathcal{F}, \mu)$ be a probability space with a filtration $\mathcal{F}_t = \{\{P_s\}, 0 \leq s \leq t\}$ adapted to the exchange rate history. Let X_t^r be the optimal demand of a rational "benchmark" trader who has the price history as his only information source and who maximizes the objective function of the financial institution.[11] The optimal foreign currency demand depends only on the expectations for the first and second moment of the excess return, that is,

$$\begin{aligned} X_t^r &= \operatorname{argmax} E\left[J^i(t) \mid \mathcal{F}_t\right] = \frac{E(dR_t \mid \mathcal{F}_t)}{\rho E(dR_t^2 \mid \mathcal{F}_t)} \\ &\equiv D\left[E(dR_t \mid \mathcal{F}_t), E(dR_t^2 \mid \mathcal{F}_t)\right]. \end{aligned}$$

We can refer to the function $D[\cdot, \cdot]$ as the rational trader's trading rule. The irrational demand component of our traders is defined as the additional noise demand that results from an ad hoc expectational error in the first moment of the excess return under the trading rule $D[\cdot, \cdot]$.

Assumption 3 (Traders). *The traders follow the trading rule $D[\cdot, \cdot]$ of a rational trader, but make expectational errors about the first moment of the excess return. Trader beliefs about the excess return deviate from those under perfect inference under price information $\mathcal{F}_t = \{\{P_s\}, 0 \leq s \leq t\}$ by a stochastic expectational error $(\Psi_t + \Psi_t^i)dt$. For a financial institution $i \in \mathcal{T} \subseteq \mathcal{I}$, the foreign currency demand is given by*

$$X_t^i = D[\cdot, \cdot] = \frac{E(dR_t \mid \mathcal{F}_t) + (\Psi_t + \Psi_t^i)dt}{\rho E(dR_t^2 \mid \mathcal{F}_t)}.$$

The error processes Ψ_t and Ψ_t^i are exogenous. The process Ψ_t is common to all traders and Ψ_t^i is idiosyncratic to trader i.

The excess return expectations of each trader can be decomposed into the perfect inference component $E(dR_t \mid \mathcal{F}_t)$ and an expectational error component $(\Psi_t + \Psi_t^i)dt$. The error component denotes the temporary over- or underestimation of the returns to foreign currency. For the special case in which the error component becomes zero, we obtain the benchmark case of fully rational traders. Assumption 3 implies that traders are generally a "mixture" of fully rational traders and pure noise traders. The term $E(dR_t \mid \mathcal{F}_t)/\rho E(dR_t^2 \mid \mathcal{F}_t)$ corresponds to the demand of a fully rational trader and $(\Psi_t + \Psi_t^i)dt/\rho E(dR_t^2 \mid \mathcal{F}_t)$ represents the noise. The common ad hoc assumption about noise traders is replaced by an ad hoc assumption about expectational errors. The expectational errors concern the first

[11] In particular, our "benchmark" rational trader has no information about the process underlying the net currency supply nor does he observe the demand of other imperfect traders. Furthermore, there are no agency problems between the trader and the financial institution.

moment of the return, whereas expectations about the second moment are always correct.

Evidence for expectational errors in the foreign exchange market comes from the widely documented forward discount bias.[12] Frankel and Froot (1990, 1993) provide direct evidence for expectational errors in survey data. Contemporaneous research attempts to derive such expectational errors from testable psychological and behavioral principles. Daniel, Hirshleifer, and Subrahmanyam (1997) argue that trader overconfidence with respect to private signals can explain a variety of market abnormalities. In their model expectational errors result from an overestimation of the precision of private signals. Pure expectational errors, like in Assumption 3, can be viewed as the limit case of overconfident trading in which, holding constant the traders' perceived signal precision, the actual signal becomes very noisy. We can thus rationalize the trading errors in our model as a closed form representation of overconfident trading.

It is crucial that the expectational errors of the traders are not independent. This is assured by assuming a common prediction error component. The common component of the expectational error creates endogenous noise in the exchange rate, which impairs the dynamic inference of all traders.[13] To keep the model framework tractable, we assume that the three stochastic processes Θ_t, Ψ_t, and Ψ_t^i follow the continuous time version of an AR(1) process. To simplify the model further, we also assume the same degree of mean reversion for all three processes. Identical mean reversion allows for a simple closed form solution of the dynamic inference problem. Assumption 4 summarizes the stochastic structure:

Assumption 4 (Stochastic Structure). *The fundamental process Θ_t and the prediction error processes Ψ_t and Ψ_t^i all follow Ornstein–Uhlenbeck processes*

$$\begin{aligned} d\Theta_t &= -a\Theta_t dt + b_\Theta dw_\Theta \\ d\Psi_t &= -a\Psi_t dt + b_\Psi dw_\Psi \\ d\Psi_t^i &= -a\Psi_t^i dt + b_{\Psi^i} dw_\Psi^i \end{aligned}$$

[12] This bias is sometimes interpreted as a risk premium. For evidence against this view, see Frankel and Froot (1989). Engel (1996) provides a recent survey on the subject.

[13] Expectational errors are necessary to prevent full revelation of the market state. However, all our results are robust to the inclusion of a (small) subset of fully rational traders without expectational errors. Assume, for example, a percentage χ are fully rational "benchmark" traders and a percentage $1-\chi$ are of the previous trader type with expectational errors. The noise component in the aggregate demand diminishes by a factor $1-\chi$, while the rational demand component remains unchanged. We can define a variable transformation $\widetilde{\Psi}_t = (1-\chi)\,\Psi_t$. Replacing the parameter b_Ψ in Assumption 4 with $\widetilde{b}_\Psi = (1-\chi)^{-1}\, b_\Psi$, the price equilibrium is still of the same form. Only the entry decision and the utility of the financial institution is modified, as it is more advantageous to hire a perfect trader (without expectational errors). But if the probability of (randomly) hiring a perfect trader is small, the institutional utility function and the entry decision is approximately correct and the entry equilibrium is not altered qualitatively.

with initial normal distributions $\Theta_0 \sim N(0, b_\Theta/2a)$, $\Psi_0 \sim N(0, b_\Psi/2a)$ *and* $\Psi_0^i \sim N(0, b_{\Psi^i}/2a)$*;* $a > 0$*. The Wiener processes* w_Ψ^i, w_Ψ, w_Θ*, and the initial distributions are stochastically independent for all* $i \in \mathcal{I}$*.*

The stochastic structure assumed here is certainly very restrictive, but it allows for a simple exposition and closed form solutions throughout the article.

2. The Exchange Rate Equilibrium

All financial institutions form the set $\mathcal{I}$, of which the subset $\mathcal{T} \subseteq \mathcal{I}$ of institutions decides to enter the market, each hiring a single trader. The (relative) size of the trader set (percentage of entrants) is defined as $\lambda \equiv \#(\mathcal{T})/\#(\mathcal{I})$, $0 < \lambda \leq 1$, where $\#(\cdot)$ denotes the number of set elements. The analysis simplifies if we focus on the limit case of infinitely many institutions and traders representing a "large" market. For this limit case we can easily characterize the linear price equilibrium for any fixed percentage λ of entrants. Section 3 then solves for the specific value λ^* which results from the competitive market entry decisions of the financial institutions. To derive the exchange rate equilibrium, we first conjecture a linear equilibrium in state vector $z_t = (\Theta_t, \Psi_t)^T$, where T indicates the transposed. Second, we solve the dynamic inference problem and obtain the optimal prediction $\widehat{z_t} = (\widehat{\Theta}_t, \widehat{\Psi}_t)^T$ of the state variables z_t. This allows us to determine the optimal excess return prediction $E(dR_t \mid \mathcal{F}_t)$. In a third step, these optimal beliefs are adjusted for the expectational error of a trader. The resulting asset demand is aggregated and the price coefficients result from the market-clearing condition.

We conjecture that the exchange rate equilibrium is linear in the net supply process Θ_t and the common prediction error Ψ_t. For price coefficients p_0, p_Θ, and p_Ψ, we assume $P_t = p_0 + p_\Theta \Theta_t + p_\Psi \Psi_t$. For a large (competitive) financial market with many traders, the idiosyncratic component of the traders' expectational errors do not influence the price process.[14]

2.1 The filtration problem

In order to predict the excess return dR_t, traders have to infer the state $z_t = (\Theta_t, \Psi_t)^T$ of the market process for a given price history. Continuous observation of the exchange rate history reveals the true state of the market process only partially. The deviation of the optimal prediction $\widehat{z_t} = E(z_t \mid \mathcal{F}_t)$ from the true state z_t of the market process is referred to as the *inference error* and is defined as $\Delta_t \equiv \widehat{z_t} - z_t = (\widehat{\Theta}_t - \Theta_t, \widehat{\Psi}_t - \Psi_t)^T$. Proposition 1 characterizes the inference error process for the conjectured exchange rate process.

[14] In a financial market with nonatomistic traders, the exchange rate equilibrium is also influenced by all idiosyncratic errors.

Proposition 1 (Optimal Inference). *Optimal dynamic inference based on observation of only the exchange rate implies an inference error $\Delta_t = \widehat{z}_t - z_t = (\widehat{\Theta}_t - \Theta_t, \widehat{\Psi}_t - \Psi_t)^T$ which follows an Ornstein–Uhlenbeck process*

$$d\Delta_t = a_{zz}\Delta_t dt + b_\Delta dw_t,$$

where $dw_t = (dw_\Theta, dw_\Psi)^T$ denotes a Wiener process and coefficients are

$$a_{zz} = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix}, \quad b_\Delta = \frac{b_\Theta b_\Psi}{p_\Theta^2 b_\Theta^2 + p_\Psi^2 b_\Psi^2} \begin{bmatrix} -p_\Psi^2 b_\Psi & p_\Theta p_\Psi b_\Theta \\ p_\Theta p_\Psi b_\Psi & -p_\Theta^2 b_\Theta \end{bmatrix}.$$

Proof. See Appendix A.

The inference error process characterizes the maximal information about the state $z_t = (\Theta_t, \Psi_t)^T$ which can in steady state (asymptotically) be obtained from optimal exchange rate inference. As both state variables follow Ornstein–Uhlenbeck processes with an identical degree a of mean reversion, we obtain a closed form solution for the inference error processes. If the noise component in the price process decreases ($b_\Psi \to 0$), then the elements of b_Δ become zero. In this case the exchange rate history reveals all information about the fundamental process Θ_t.

2.2 Asset demand and market clearing

Proposition 1 describes the beliefs of a trader with perfect inference abilities about the state of the financial market. The expected excess return under rational expectations depends on the predicted state $\widehat{z}_t = (\widehat{\Theta}_t, \widehat{\Psi}_t)^T$ of the market process and is such that

$$E(dR_t \mid \mathcal{F}_t) = E(dP_t - \overline{r}P_t dt \mid \mathcal{F}_t) = \left(e_0 + e_\Theta \widehat{\Theta}_t + e_\Psi \widehat{\Psi}_t\right) dt \qquad (1)$$

with coefficients e_0, e_Θ, and e_Ψ given in Appendix A. By Assumption 3, traders make *expectational errors* relative to the optimal excess return beliefs. The excess return expectation of any trader is disturbed by the stochastic, mean zero, expectational error $(\Psi_t + \Psi_t^i)dt$.

The model assumes a constant opportunity cost of investment $\overline{r}$. The instantaneous volatility of the excess return process is therefore identical to the price volatility of the foreign currency and is given by

$$E(dR_t^2 \mid \mathcal{F}_t) = \left(p_\Theta^2 b_\Theta^2 + p_\Psi^2 b_\Psi^2\right) dt \equiv V dt. \qquad (2)$$

Trader demand depends on the expected excess return, the instantaneous volatility of the excess return V, and the risk aversion parameter ρ and follows as

$$X_t^i(p_0, p_\Theta, p_\Psi) = \frac{E(dR_t \mid \mathcal{F}_t) + (\Psi_t + \Psi_t^i)dt}{\rho V dt}. \qquad (3)$$

The next step is to aggregate the individual demand functions. For a large financial market with many financial institutions and traders, we can use the law of large numbers and discard the independent idiosyncratic demand components. Formally, we define a charge space $(\mathcal{I}, \mathcal{P}(\mathcal{I}), \mu)$ where $\mathcal{P}(\mathcal{I})$ is the collection of subsets of $\mathcal{I}$, and $\mu : \mathcal{P}(\mathcal{I}) \to \Re_+$ is a finitely additive measure. Let the set of natural numbers represent the set of potential entrants $\mathcal{I} = \{1, 2, \ldots, N\}$.[15] A measure (charge) is given by $\mu(A) = \lim_{N\to\infty} \frac{1}{N} \#(A \cap \{1, 2, \ldots, N\}), \forall A \in \mathcal{I}$, where $\#(\cdot)$ denotes the number of set elements. For an indicator function $\mathrm{IN}(\cdot)$, we can then state the aggregate demand of a set $\mathcal{T} \subseteq \mathcal{I}$ of entrants as

$$\int_{i\in\mathcal{T}} X_t^i d\mu(i) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} X_t^i \,\mathrm{IN}(i \in \mathcal{T}).$$

It is instructive to examine the aggregate noise that is generated by erroneous trader beliefs. For a large financial market ($N \to \infty$), the aggregate demand is affected only by the common prediction errors. Given the size $\lambda = \mu(\mathcal{T})$ of the trader set, we obtain an aggregate endogenous noise level

$$\int_{i\in\mathcal{T}} \frac{\Psi_t + \Psi_t^i}{\rho V} d\mu(i) = \left(\frac{\rho}{\lambda} V\right)^{-1} \Psi_t. \tag{4}$$

The endogenous noise in the price process increases in the size of the trader set λ and decreases in the volatility V of the price process. Entry decisions exercise a negative information externality on other traders by increasing the noise in the exchange rate. More volatility decreases the endogenous noise. Under higher volatility, risk-averse traders reduce their demand and this moderates the negative effect of expectational errors on the amount of endogenous noise.

The endogenous noise formulation can be contrasted with previous noisy rational expectation models. The latter assume a constant exogenous noise shock or a stationary noise process in their dynamic extension.[16] In our model the total noise is inversely related to the term $\frac{\rho}{\lambda} V$, which denotes the exchange rate volatility adjusted by the ratio of the risk aversion ρ and the size of the trader set λ. The ratio $\frac{\rho}{\lambda}$ measures the collective risk aversion of the market. We can interpret $\frac{\rho}{\lambda} V$ as the average exchange rate risk per trader of a unit of net foreign currency supply. It is henceforth referred to as the *market risk*. Higher market risk decreases the endogenous noise and decreases a trader's demand for foreign currency.

[15] See He and Wang (1995) for a similar approach. For a reference to charge theory and its application to σ fields, see Rao and Rao (1983). For a general discussion of the aggregation problem, see Feldman and Gilles (1984).

[16] For an example of a dynamic noisy rational expectations model, see Wang (1993).

Finally, we aggregate the individual demand functions to the total speculative demand X_t and impose the market-clearing condition:

$$X_t = \int_{i \in \mathcal{T}} X_t^i(p_0, p_\Theta, p_\Psi) d\mu(i) = \Theta_t + \gamma \left[P_t(p_0, p_\Theta, p_\Psi) - \overline{P} \right]. \quad (5)$$

The market-clearing condition of Equation (5) together with Equations (1)–(4) determines the three price coefficients. Proposition 2 characterizes the price equilibrium.

Proposition 2 (Exchange Rate Equilibrium). *A linear exchange rate equilibrium for a trader set of size λ exists if $(a + \overline{r})^2 > 2b_\Psi b_\Theta \frac{\rho}{\lambda}$. It is given by*

$$P_t = p_0 + p_\Theta \Theta_t + p_\Psi \Psi_t$$

with parameters

$$p_0 = \frac{\frac{\rho}{\lambda} V\gamma}{\overline{r} + \frac{\rho}{\lambda} V\gamma} \overline{P}, \quad p_\Theta = -\frac{\frac{\rho}{\lambda} V}{a + \overline{r} + \frac{\rho}{\lambda} V\gamma}, \quad p_\Psi = \frac{1}{a + \overline{r} + \frac{\rho}{\lambda} V\gamma}, \quad (6)$$

and the (instantaneous) volatility $V = p_\Theta^2 b_\Theta^2 + p_\Psi^2 b_\Psi^2$ characterized by[17]

$$G(V, \lambda) \equiv \left(a + \overline{r} + \frac{\rho}{\lambda} V\gamma \right)^2 - \left(\frac{\rho}{\lambda} \right)^2 V b_\Theta^2 - \frac{b_\Psi^2}{V} = 0. \quad (7)$$

Proof. See Appendix B.

To interpret Proposition 2, it is helpful to look at the expected excess return $E(dR_t)$ of foreign currency as implied by the exchange rate process P_t. For the special case of a completely price inelastic net supply ($\gamma = 0$) and identical riskless rates in both countries ($\overline{r} = 0$), we get

$$E(dR_t) = \left(\frac{\rho}{\lambda} V \Theta_t - \Psi_t \right) dt. \quad (8)$$

The expected excess return $E(dR_t)$ in a time interval dt is proportional to the product of market risk $\frac{\rho}{\lambda} V$ and net supply of foreign currency Θ_t. A larger net supply means that each trader has to hold more of the risky asset on average and the return has to increase. The common expectational error Ψ_t only shifts the expected excess return process. If traders collectively overestimate the return from holding currency ($\Psi_t > 0$), its return is low because of a high currency price. The coefficients p_0 and p_Ψ decrease as the net supply becomes more price elastic ($\gamma > 0$). A financial market with a more price elastic net supply requires lower risk premia.

[17] For a small price elasticity γ of the excess supply, Equation 7 characterizes a high and a low volatility equilibrium. A sufficient condition for a unique linear price equilibrium is $\gamma > \rho b_\Theta^2 / 2\lambda(a + \overline{r})$.

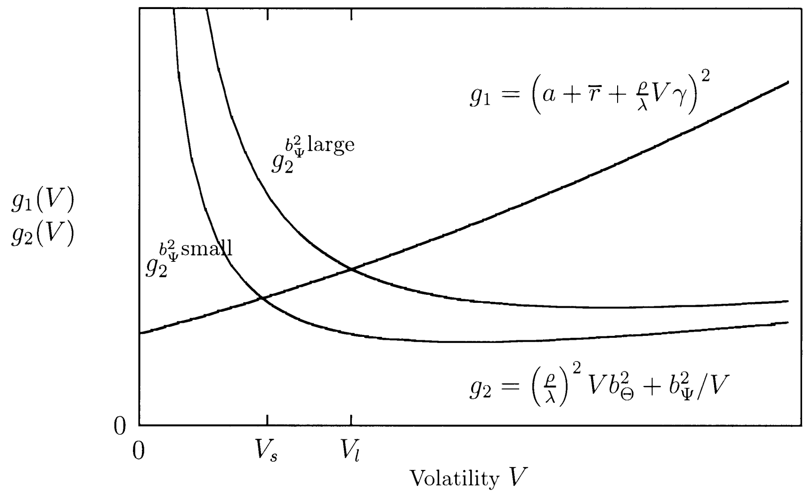

Figure 1
Exchange rate volatility for exogenous percentage λ of entry
Volatility decreases as the traders' expectational error becomes small ($b_\Psi \to 0$).

The polynomial in Equation (7) generally determines a low and a high volatility equilibrium. But if the price elasticity γ of the excess demand is sufficiently high, a unique price equilibrium is obtained. In this case we can explore the limit case of the exchange rate equilibrium as the traders become perfect in their inference abilities. Figure 1 graphs the first component $g_1 = (a + \overline{r} + \frac{\rho}{\lambda} V\gamma)^2$ and the second component $g_2 = \left(\frac{\rho}{\lambda}\right)^2 V b_\Theta^2 - b_\Psi^2/V$ of Equation (7). The second component is plotted for a small and a large expectational error parameter b_Ψ^2. The intersection of both components determines the volatility V. As the exogenous expectational error converges to zero ($b_\Psi \to 0$), the g_2 schedule shifts to the left and the volatility V and the market risk $\frac{\rho}{\lambda} V$ become zero. The exchange rate is equal to one at all times and the intertemporal profit opportunities are fully exploited. This shows that expectational errors are the only market distortion in our model.

Before we discuss the role of market entry on volatility, we explore the relationship between market risk and the informational properties of the exchange rate equilibrium in Section 2.3. This allows for a better economic interpretation of the relationship between market entry and volatility in Section 2.4.

2.3 Information content of the exchange rate

Financial institutions earn profits as risk premia for intertemporal trading of the net supply governed by the process Θ_t. Higher predictability of the net supply fluctuations, that is, a lower inference error $\widehat{\Theta}_t - \Theta_t$, allows for more informed trading. Definition 1 states the information content of the exchange rate history with respect to the fundamental process Θ_t.

Definition 1. *The information content (IC) of the exchange rate equilibrium denotes the conditional precision of the inference error $\Delta_\Theta = \widehat{\Theta}_t - \Theta_t$. Formally*

$$IC \equiv \frac{1}{E\left[(\widehat{\Theta}_t - \Theta_t)(\widehat{\Theta}_t - \Theta_t) \mid \mathcal{F}_t\right]} \quad \textit{with} \quad \widehat{\Theta}_t = E\left[\Theta_t \mid \mathcal{F}_t\right].$$

Let $\mathrm{var}(\Theta_t) = b_\Theta^2/2a$ and $\mathrm{var}(\Psi_t) = b_\Psi^2/2a$ denote the unconditional variance of the processes Θ_t and Ψ_t, respectively. The information content of the exchange rate is characterized in Proposition 3.

Proposition 3. *The information content of the exchange rate is given by*

$$IC = \left(\frac{\rho}{\lambda}V\right)^2 \frac{1}{\mathrm{var}(\Psi_t)} + \frac{1}{\mathrm{var}(\Theta_t)}. \tag{9}$$

Proof. See Appendix B.

The information content of the exchange rate is proportional to the ratio of the squared market risk and the unconditional variance of the common expectational error. Why does the information content of the exchange rate increase in the market risk? Equation (4) tells us that higher market risk reduces the endogenous noise in the exchange rate as traders become more cautious about submitting large market orders. A reduction of the endogenous noise leads to a more informative exchange rate. The higher information content is also reflected in the price parameters. Equation (6) shows that higher market risk increases the absolute value of the parameter p_Θ relative to the parameter p_Ψ as

$$\frac{\mid p_\Theta \mid}{p_\Psi} = \frac{\rho}{\lambda}V. \tag{10}$$

Net supply shocks have a greater relative exchange rate impact for a higher market risk due to a larger parameter $\mid p_\Theta \mid$ relative to the noise of the common expectational error which enters through the parameter p_Ψ. Therefore net supply shocks are easier to identify and become more distinguishable from the noise component in the exchange rate. This allows for a more precise inference of the fundamental supply process Θ_t.

With this intuition for the relationship between market risk and the information content of the price, we can now give a more meaningful interpretation to the exchange rate equilibrium characterized by Equation (7).

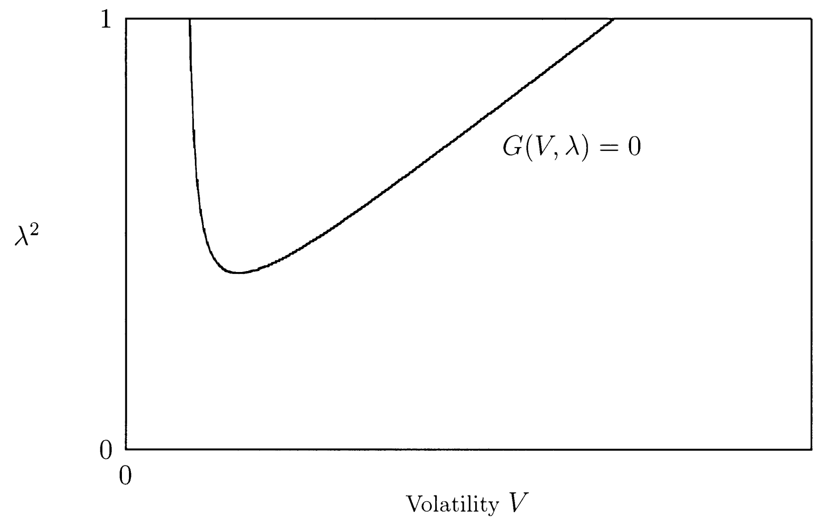

Figure 2
Potential exchange rate equilibria in the (λ^2, V) space given by $G(\lambda, V) = 0$
The percentage of entering traders is λ and V denotes the exchange rate volatility.

2.4 Risk sharing versus information externality

The following section provides the intuition for how market entry affects volatility. To simplify the algebra, we look at the special case of a fully price inelastic net supply, where $\gamma = 0$. Equation (7) can then be solved for square size (λ^2) of the trader set,

$$G(V,\lambda) = 0 \quad \Leftrightarrow \quad \lambda^2 = \frac{\rho^2 V^2 b_\Theta^2}{(a+\bar{r})^2\, V - b_\Psi^2}.$$

Figure 2 graphs the combinations for λ^2 of the trader set and the instantaneous volatility V which are consistent with a dynamic exchange rate equilibrium. The equilibrium schedule is U-shaped. In the left branch of the schedule, market entry of additional traders decreases volatility, while in the right branch entry increases exchange rate volatility.

To explain the ambiguous effect of market entry on volatility, two different effects must be distinguished. More traders increase the ability of the market to absorb the exogenous supply shocks. The collective risk aversion $\frac{\rho}{\lambda}$ of the market decreases and thus reduces exchange rate volatility. This effect can be characterized as the *risk-sharing effect*. But more traders also increase the endogenous noise expressed in Equation (4). Under increased endogenous noise, traders find it more difficult to predict the excess return. Entry by a marginal trader thus creates a *negative information effect* on traders already operating in the market. The negative information effect dominates the risk-sharing effect if the market risk $\frac{\rho}{\lambda}V$ is large. For a large market risk, the endogenous noise is small [Equation (4)] and the exchange rate history is very informative about the net supply process Θ_t [Equation (9)]. For an informative exchange rate, the negative information effect of the marginal trader is more important than the risk-sharing effect. To obtain market clearing after entry of a marginal trader, the information content of the exchange rate has to increase, which requires a volatility increase. For a low market risk and a low information content, the risk-sharing effect dominates the negative information externality. Improved risk sharing by the marginal trader makes exchange rates too informative for market clearing unless volatility decreases.

The above result contrasts with the common finding in noisy rational expectations models where entry always decreases volatility. In these models, the risk-sharing effect is the only market externality. A decrease in the market risk aversion $\frac{\rho}{\lambda}$ increases the capacity of the market to absorb the exogenous risk. In our framework, speculative trading is not mere absorption of exogenous risk, but also a source of risk through expectational errors about the return. The marginal effect of entry depends on the informational characteristics of the exchange rate equilibrium. A financial market with uninformative prices benefits from entry of traders who improve risk sharing. Entry is price stabilizing. As exchange rates become more informative, the negative information effect tends to dominate and speculation by additional traders becomes destabilizing.

3. The Market Entry Equilibria

The discussion of the exchange rate equilibrium was a partial equilibrium analysis since it assumed an exogenous set of traders. We now turn to the case of endogenous entry and derive the trader set from the available profit opportunities and the traders' expectational errors. The optimal entry strategy for a financial institution is characterized in Section 3.1. Sections 3.2 and 3.3 analyze the competitive entry equilibria and their stability. Market efficiency is discussed in Section 3.4.

3.1 Optimal hiring decisions

Financial institutions make simultaneous entry decisions at time $t = 0$. The entry equilibrium is a Nash equilibrium, in which each institution makes an optimal hiring decision based on rational ex-ante expectations about the profit flow generated by a trader. The entry equilibrium is characterized by the percentage $\lambda^* = \mu(\mathcal{T}^*)$ of financial institutions that decide to enter and hire a trader. For a simpler exposition of the results, we concentrate on the case where the net supply is completely price inelastic ($\gamma = 0$) and there is a zero differential in the riskless rates ($\overline{r} = 0$).[18]

Financial institutions hold rational expectations about the first and second moments of a trader's profit flow. The aggregate trading profits of the institution Π_t^i follow an Ito process which depends on the state vector $v_t^i = (\Theta_t, \Psi_t, \Psi_t^i)$. The profit flow evolves according to

$$d\Pi_t^i = \begin{cases} \left[a_\Pi(v_t^i) - c(.)\right] dt + b_\Pi(v_t^i) dw_t & \text{for} \quad y^i = 1 \\ 0 & \text{for} \quad y^i = 0 \end{cases}, \tag{11}$$

where the average gross profit flow from trading in dt is $a_\Pi(v_t^i)dt$ and the stochastic component of the profit increment is given by $b_\Pi(v_t^i)dw_t$. Both the mean and the variance of the profit flow depend on the state vector v_t^i and change over time. The currency position of a trader follows as

$$X(v_t^i) = \frac{1}{\lambda}\Theta_t + \frac{1}{\rho V}\Psi_t^i.$$

The idiosyncratic expectational error Ψ_t^i creates a heterogeneous currency demand around an otherwise uniform distribution given by each trader's share $\frac{1}{\lambda}\Theta_t$ of the excess supply.

The financial institution has to find a dual decision rule $y^i(V, \lambda) \in \{0, 1\}$, which may depend on the instantaneous volatility V and size λ of the set of entrants, solving the optimization problem

$$J^i(V, \lambda) = \max_{\{y(V,\lambda)\}} E \int_{t=0}^{\infty} e^{-rt} \left[d\Pi_t^i - \tfrac{1}{2}\rho \left(d\Pi_t^i\right)^2 \right] \tag{12}$$

for a profit flow given by Equation (11). The solution to the institutional optimization problem is provided by Proposition 4:

Proposition 4 (Optimal Market Entry). *The value function to the financial institution is given by*

$$J^i(V, \lambda) = \frac{1}{r} y(V, \lambda) F(V, \lambda).$$

The expected utility flow

$$F(V, \lambda) \equiv \widehat{a}_\Pi - c(\cdot) - \frac{1}{2}\rho \widehat{b_\Pi^2}$$

[18] Solutions simplify for $\gamma = 0$, as traders do not face the adverse selection problem that collective expectational errors imply for a price elastic net supply process.

has parameters $\widehat{a}_{\Pi} = E\left[a_{\Pi}(v^i)\right]$ *and* $\widehat{b^2_{\Pi}} = E\left[b_{\Pi}(v^i)b_{\Pi}(v^i)^T\right]$. *The optimal hiring policy for any financial institution is*

$$y(V, \lambda) = \begin{cases} 1 & \textit{for} \quad F(V, \lambda) > 0 \\ 0 & \textit{for} \quad F(V, \lambda) \leq 0 \end{cases} .$$

For $\gamma = 0$ *the expected utility flow becomes*

$$F(V, \lambda) \equiv \frac{1}{4a\lambda}\left[\frac{\rho}{\lambda}Vb^2_{\Theta} - \frac{1}{\frac{\rho}{\lambda}V}b^2_{\Psi^i} - 4a\lambda c(.)\right]. \tag{13}$$

Proof. See Appendix C.

A financial institution enters if $F(V, \lambda) > 0$ and does not enter if $F(V, \lambda) < 0$. It is indifferent between both options for $F(V, \lambda) = 0$. The expected utility flow $F(V, \lambda)$ increases in the unconditional net profit expectations $\widehat{a}_{\Pi} - c(\cdot)$ and decreases in the unconditional volatility expectations $\widehat{b^2_{\Pi}}$. Higher risk aversion of the financial institution requires a higher expected trading profit to maintain the same entry threshold.

The entry threshold $F(V, \lambda) = 0$ depends on the volatility V and the size λ of the trader set as stated by Equation (13). For zero operating costs $[c(\cdot) = 0]$, we can rewrite the entry threshold as

$$\frac{\rho}{\lambda}V = \frac{b_{\Psi^i}}{b_{\Theta}}.$$

Financial institutions are indifferent between entry and no entry if the market risk is equal to the ratio of the volatility parameter b_{Ψ^i} of the idiosyncratic expectational error and the volatility parameter b_{Θ} of the net supply. As shown by Equation (8), the excess return on the asset is proportional to the market risk $\frac{\rho}{\lambda}V$. If the idiosyncratic expectational error is high relative to the volatility of the net supply, and if risk premia are low ($\frac{\rho}{\lambda}V < b_{\Psi^i}/b_{\Theta}$), financial institutions do not enter the market. If, on the other hand, the idiosyncratic error of the traders is small and the risk premia are high, financial institutions prefer entry. For institutions to be indifferent about entry, the market risk and the risk premia have to be just high enough to compensate the financial institutions for the risk of erroneous trading by their trader.

The common expectational error of a trader does not constitute any risk for the financial institution under a completely price inelastic net supply. Common expectational errors do not subject the trader to any adverse selection problem since all traders have erroneous demands and none of the traders acquires a net position. For the more general case of a price elastic net supply ($\gamma > 0$), common expectational errors expose traders to an adverse (loss making) position taking.[19]

3.2 Market entry and volatility

Having characterized the optimal entry strategies for financial institutions, we now address the core question of our inquiry. Are the expectational errors internalized in the entry decisions of the financial institutions? Is the competitive equilibrium characterized by an optimal number of entrants?

To determine the competitive entry equilibria, we examine the set of potential exchange rate equilibria from Section 2.4 for combinations (λ^2, V) that keep financial institutions from reversing their entry strategy. It is straightforward to solve the entry indifference curve for λ^2 of the trader set,

$$F(V, \lambda) = 0 \quad \Leftrightarrow \quad \lambda^2 = \frac{\rho^2 V^2 b_\Theta^2}{4a\rho c(\cdot)V + b_{\Psi^i}^2}.$$

The solution to the entry equilibrium generally depends on the operating costs $c(\cdot)$. For simplicity, we first look at the benchmark case of constant operating costs $c(\cdot) = c$. Figure 3 plots both the exchange rate equilibrium schedule and the entry indifference curve. The entry indifference curve increases with volatility. Higher volatility can sustain more financial institutions in the market. Financial institutions prefer market entry for all combinations (λ^2, V) below and to the right of the curve. No entry is preferred for points above and to the left of the curve. Higher operating costs shift the curve downward as fewer financial institutions can expect to recover their operating costs for any given volatility. Similarly, a higher idiosyncratic trading error (increase in b_{Ψ^i}) deters financial institutions from entry and shifts the entry indifference curve downward.

The U-shaped price equilibrium schedule graphs all combinations (λ^2, V) consistent with a linear price equilibrium. Its shape was already discussed in Section 2.4. Market entry decreases volatility for a low volatility level in the left branch of the graph. For low volatility levels, market risk is low and prices are relatively uninformative about the net supply process. The risk-sharing effect dominates the negative information effect for additional entry. For high volatility levels and high market risk in the right branch of the graph, the negative information externality dominates the risk-sharing effect. Additional entry increases exchange rate volatility.

The $F(V, \lambda) = 0$ locus intersects the $G(V, \lambda) = 0$ locus in its right branch if the ratio of the idiosyncratic and the common expectational error parameters is larger than one, or $b_{\Psi^i}/b_\Psi > 1$. Higher idiosyncratic or

[19] Our further analysis neglects this adverse selection risk by focusing on a completely price inelastic net supply.

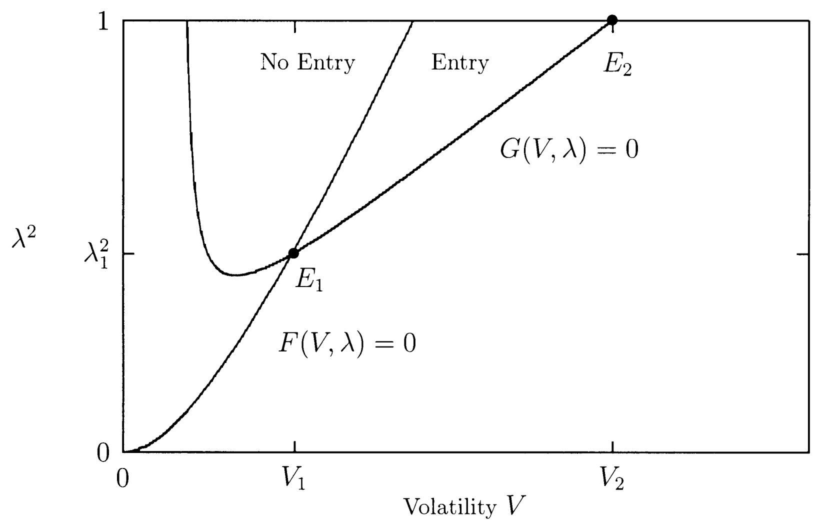

Figure 3
Low volatility E_1 and high volatility E_2 equilibria under endogenous entry
The percentage of entering traders is λ and V denotes the exchange rate volatility. $F(\lambda, V) = 0$ marks the indifference curve between entry and no-entry. Equilibrium E_1 is unstable and equilibrium E_2 is stable.

institutional risk requires higher volatility and therefore higher risk premia to induce entry. Note also that a higher operating cost c will tend to yield an intersection in the right branch of the $G(V, \lambda) = 0$ locus.[20]

3.3 Equilibria and Equilibrium Stability

Figure 3 shows two different financial market equilibria. The first equilibrium, denoted E_1, follows as the intersection of the exchange rate schedule with the entry indifference curve. This equilibrium is characterized by partial entry of $\lambda_1^* < 1$ financial institutions. Financial institutions are indifferent between entry and no entry. The institutional utility is zero. A second equilibrium, denoted E_2, is the corner solution where all financial institutions enter the market. This second equilibrium implies positive institutional utility. It is associated with higher volatility. For equilibrium E_2, the negative information externality dominates the risk absorption effect. Excessive entry leads to an increase in endogenous noise and high volatility. Financial institutions do not internalize the risk and information externality of their entry decisions on other speculators. With competitive entry decisions, they mutually contribute to the destabilization of the exchange rate and secure high-risk premia for intertemporal demand mediation. The

[20] However, operating costs must not be too large. Above a certain threshold $\overline{c}$, no entry equilibrium exists.

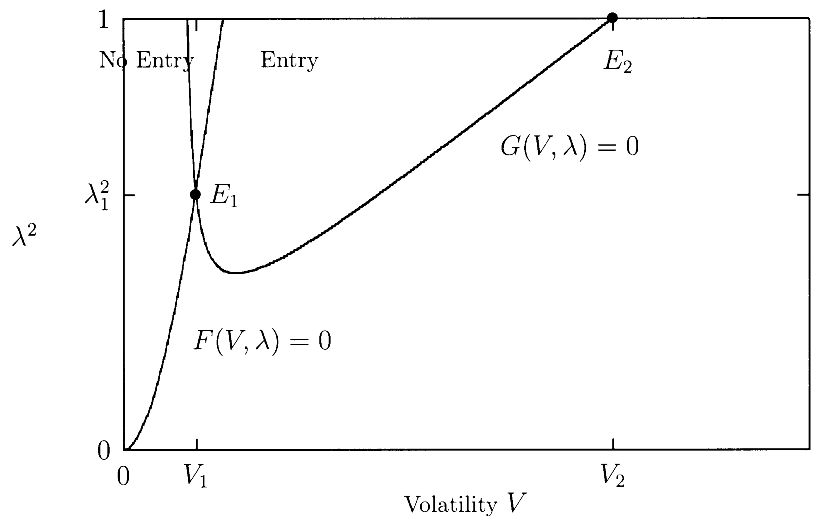

Figure 4
Two stable market equilibria under endogenous entry
Equilibrium E_1 has low volatility and equilibrium E_2 has high volatility.

risk premia in turn distort their entry incentives, making excessive entry a profitable strategy.

An important difference between both equilibria is their stability. Consider a combination (λ^2, V) on the price equilibrium schedule slightly to the right of equilibrium E_1. At this point all entering institutions have positive utility and are strictly better off than financial institutions which did not enter. Their best response would be entry. This adjustment brings us directly to equilibrium E_2. The low volatility equilibrium E_1 is unstable. By a similar argument, any deviation to the left of E_2 brings us back to E_2. The high volatility equilibrium is stable.

Alternatively, the $F(V, \lambda) = 0$ locus may intersect the left branch of the $G(V, \lambda) = 0$ locus as shown in Figure 4. This requires that the idiosyncratic expectational error is small relative to the common error, or $b_{\Psi^i}/b_\Psi < 1$. It is easy to verify that now both the low and the high volatility equilibria are stable and they may both persist in the financial market. Such persistence of high or low volatility conditions are commonly observed in financial markets. In Section 4.2, we explore the implication of exchange rate heteroscedasticity for the trading profits of the financial institutions.

Generally, operating costs $c(\cdot)$ need not be constant for a different number of entrants. Financial institutions may face increasing operating costs as more entry occurs. In this case the $F(V, \lambda) = 0$ locus will become S-shaped, as illustrated in Figure 5. Entry requires higher volatility and higher risk

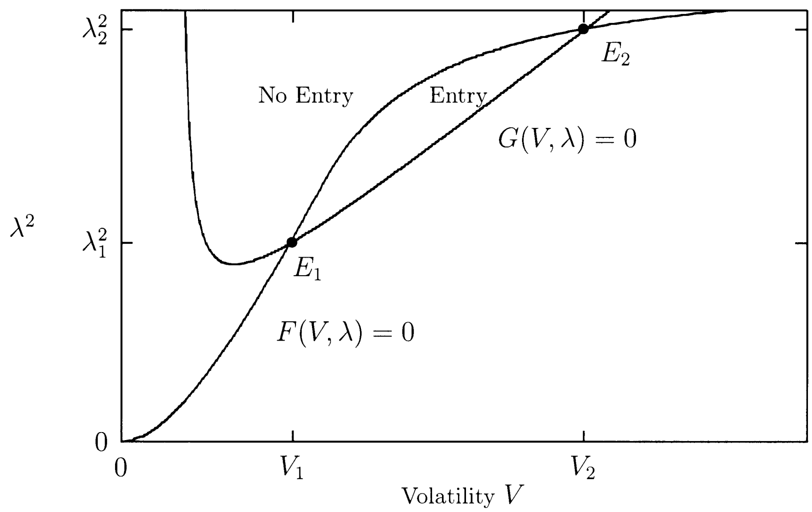

Figure 5
Two market equilibria with partial entry for costs increasing in trader participation λ

premia as a compensation for the increasing operating costs in situations of excessive market entry. Both the high and the low volatility equilibrium are now characterized by partial entry.

3.4 Allocational efficiency and taxation

We can compare the competitive entry equilibrium with the efficient solution that a central planer would implement. The efficient solution has to rely on traders with the same expectational errors. The set of feasible solutions is therefore traced by the $G(V, \lambda) = 0$ locus. Since all gains of the financial institutions are paid for by agents with the exogenous trading needs represented by the net supply function, a reasonable objective of a central planer might just be to minimize the aggregate operating costs of the market. The minimum point (λ_e^2, V_e) of the $G(V, \lambda) = 0$ locus represents the efficient exchange rate equilibrium. Moving up on either the left or the right branch of the exchange rate equilibrium implies more traders and higher aggregate operating costs.

This efficiency criterion does not account for the potential welfare loss from volatility itself. But even by the cost minimization criterion, the stable high volatility equilibrium E_2 in Figure 3 is allocationally less efficient than the unstable low volatility equilibrium E_1. In the high volatility equilibrium, the financial sector extracts higher rents as risk premia for intertemporal net supply mediation from the nonfinancial sector. The financial sector therefore

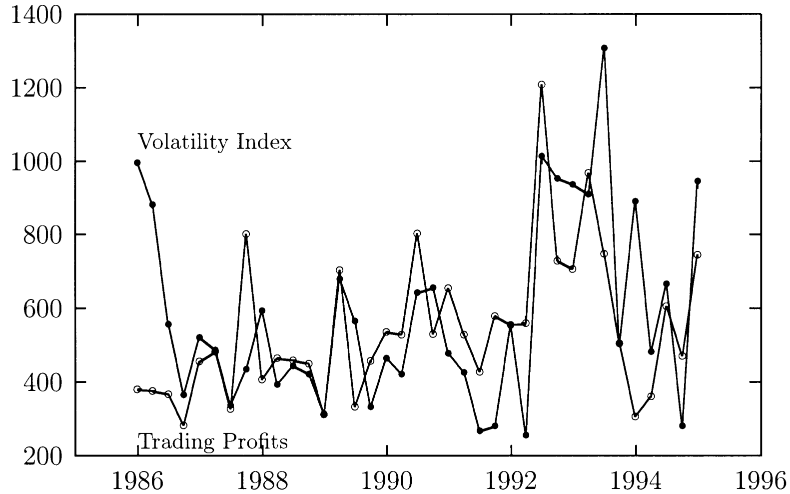

Figure 6
Aggregate FX trading profits (in millions of U.S. dollars) of the 20 largest U.S. banks and a global foreign exchange volatility index

benefits from the information externality of imperfect trading that maintains high-risk premia.

Ito, Lyons, and Melvin (1998) document that Tokyo traders were legally restricted from lunch-hour trading and that the suspension of this rule in 1993 doubled lunch-hour return volatility. If we represent the Tokyo traders by the set $[\overline{\lambda}, \lambda_2]$, their temporary market exit implies a constrained entry set $[0, \overline{\lambda}]$ with an equilibrium on the $G(V, \lambda) = 0$ locus below E_2 (Figure 5). The constrained entry equilibrium is characterized by lower volatility. Suspension of the lunch-hour rule implies a volatility increase to the level V_2. Our model can therefore explain the Tokyo evidence.

Taxation of financial institutions may alleviate the externality problem and increase allocational efficiency. A tax, which increases operating costs, shifts the $F(V, \lambda) = 0$ locus in Figure 5 downwards. The stable high volatility equilibrium shifts to the left on the $G(V, \lambda) = 0$ locus. However, beyond a certain tax rate, entry is not profitable enough and the financial market no longer exists. We emphasize that self-regulation of the financial sector cannot solve the market failure. The high volatility equilibrium creates a greater income redistribution from the nonfinancial to the financial sector. It is not in the interest of the financial sector to restrict these rents by restricting market entry.

4. Additional Empirical Implications

The previous section showed that competitive market entry under imperfect predictive abilities can explain excess volatility in financial markets. In Section 4.1, we examine the model's implications for the volume puzzle. Section 4.2 looks at the correlation between trading profits and volatility in the foreign exchange market.

4.1 Trading volume

The volume implications of information-based trading have recently drawn greater attention, including the volume composition in financial markets. The Bank for International Settlements estimated in April 1995 that the daily global turnover in the foreign exchange market amounted to U.S. \$1.2 trillion. The exogenous trading demand appears to be much smaller. Frankel (1996) calculates that only 30.6% of the volume is trading for customers. Can our framework explain large intramarket turnover between traders relative to the total trading volume?

To examine this question, we calculate the total trading volume in the interval dt

$$\begin{aligned} d\overline{Vol}^{total} &= \tfrac{1}{2}E\left[\mid d\Theta \mid + \int_{i\in\mathcal{T}} \mid dX^i \mid d\mu(i)\right] \\ &= \tfrac{1}{2}\sqrt{\tfrac{2}{\pi}b_\Theta^2 dt} + \tfrac{1}{2}\sqrt{\tfrac{2}{\pi}\left[b_\Theta^2 + \left(\frac{\rho}{\lambda}V\right)^{-2} b_{\Psi^i}^2\right]dt}. \end{aligned}$$

The term $\mid d\Theta \mid$ denotes the trading volume from exogenous supply shocks and the second term accounts for the aggregated trading volume of the traders. Premultiplication by $\frac{1}{2}$ eliminates double counting of transactions. The term $(\frac{\rho}{\lambda}V)^{-2}b_{\Psi^i}^2$ characterizes the intramarket trading component caused by idiosyncratic prediction errors. We can express the percentage of the volume generated by the trading between traders relative to the total trading volume as

$$\frac{d\overline{Vol}^{intra}}{d\overline{Vol}^{total}} = \frac{\left(\frac{\rho}{\lambda}V\right)^{-1} b_{\Psi^i}}{b_\Theta + \sqrt{b_\Theta^2 + \left(\frac{\rho}{\lambda}V\right)^{-2} b_{\Psi^i}^2}}. \tag{14}$$

For zero operating costs we have $\left(\frac{\rho}{\lambda}V\right)^{-1} b_{\Psi^i} = b_\Theta$ and Equation (14) simplifies to

$$\frac{d\overline{Vol}^{intra}}{d\overline{Vol}^{total}} = \frac{1}{1+\sqrt{2}} \approx .41.$$

For this benchmark case our model predicts that approximately 41% of the transactions should be between traders.[21] This is less than the 70% estimated by Frankel for the global foreign exchange market. This difference finds a plausible explanation in the decentralized structure of the foreign exchange market. Lyons (1997b) emphasizes intermediary transactions as an important source of intramarket trading volume. If we assume that the exogenous supply shock is passed on consecutively to $n \geq 1$ different traders before it is disseminated to the entire market, the intramarket trading share increases to

$$\frac{d\overline{Vol}^{intra}}{d\overline{Vol}^{total}} = \frac{1 + \sqrt{2} + 2(n-1)}{1 + \sqrt{2} + 2n}.$$

For example, $n = 2$ implies an intramarket trading volume of approximately 69% of total trading. The decentralized market structure of the foreign exchange market appears important for explaining the observed intramarket volume.

For the benchmark solution, the percentage of intramarket trading volume does not depend on any of the model parameters. Therefore trading volume does not collapse even as traders become arbitrarily rational ($b^2_{\Psi^i} = b^2_{\Psi} \to 0$). Surprisingly, the equilibrium does not converge to a situation where all trading results from exogenous net supply shocks. Increasing trader rationality decreases the set of entrants and increases the market's collective risk aversion. As price volatility decreases, so does the risk-sharing capacity of the market. The endogenous noise and the market risk $\frac{\rho}{\lambda}V$ remain constant as $V \to 0$ and $\frac{\rho}{\lambda} \to \infty$. Each trader turns over a larger volume as the size of the trader set decreases. The ratio of intramarket trading to total market volume remains constant.

4.2 Trading profits under heteroscedasticity

Various financial markets, including the foreign exchange market, are characterized by price heteroscedasticity. Figure 4 provides an example in which both the low and the high volatility equilibria are stable and may therefore be persistent. So far we assumed that entry decisions are taken only once at the beginning of a trading process that continues over an infinite horizon. We may alternatively restrict the trading period to a finite interval after which the entry game is repeated.[22] This setting can generate a heteroscedastic price history.

The high and the low volatility equilibria may alternate in the different stage games. They are associated with different expected trading profits for the financial institutions. The unconditional expected gross trading profits

[21] Note that equilibria with higher market risk have lower intramarket volume.

[22] The equilibria are independent of the time horizon over which trading is conducted.

Table 1
Trading Profits and Foreign Exchange Rate Volatility

Parameter	2SLS estimate	t value
β_0	776.1	9.36
β_1	-107.6	-3.12

Adjusted $R^2 = .239$, DW $= 2.17$, $\widehat{\rho} = .31$

$$\frac{\Delta\overline{\Pi}_t}{\Delta t} = \beta_0 + \beta_1 V_t^{-1} + \epsilon_t$$
$$\epsilon_t = \rho\epsilon_{t-1} + \mu_t$$

are given by

$$E\left(\frac{d\Pi_t}{dt}\right) = E\left(\frac{X_t dR_t}{dt}\right) = \frac{(a+\overline{r})^2}{2a\rho} - \frac{b_\Psi^2}{2a\rho}V_t^{-1}, \tag{15}$$

where we use $G(V,\lambda) = 0$ and assume $\gamma = 0$.

The expected trading profits decrease in the inverse V_t^{-1} of the volatility. A foreign exchange market with higher volatility implies higher trading profits. Equation (15) can be estimated for data on the trading profits of financial institutions and a measure of exchange rate volatility. In particular we can test for the sign of the coefficients under time-varying volatility V_t.

Data on the quarterly foreign exchange trading profits of U.S. financial institutions has been collected by the Federal Reserve since 1986. We obtained data on the foreign exchange trading profits for the 20 largest U.S. banks.[23] Average quarterly trading profits $\Delta\overline{\Pi}_t/\Delta t$ are used as the dependent variable. As a measure of global exchange rate volatility, we use the average volatility of the trade weighted exchange rates of the G7 industrial countries.[24] Figure 6 shows both the aggregate bank profits and the global exchange rate volatility index V_t from the first quarter of 1986 to the first quarter of 1995.

We estimate Equation (15) in a two-step Cochrane–Orcutt procedure which allows for serial correlation of the errors. The results are presented in Table 1. The regression coefficient β_0 is significant with a t value of 9.36 and the coefficient β_1 is negative with a t value of -3.12. As predicted by the model, periods of high volatility are associated with higher trading profits for financial institutions.[25]

[23] Quarterly profit data on large foreign trading banks were not available from the Federal Reserve, which collects profit data only from U.S. institutions.

[24] Daily trade-weighted exchange rate indices for the G7 countries are compiled by the Bank of England and were obtained from Datastream.

[25] Evidence on a positive profit-volatility correlation for a single market maker in the foreign exchange market is provided by Lyons (1997a), who also attempts a breakdown of profits between speculative profits and intermediation profits.

5. Conclusions

This article develops a theory of competitive market entry into a dynamic foreign exchange market. We examine the consequences of expectational errors of traders for the competitive entry decisions of financial institutions. Our model shows that expectational errors tend to create excessive market entry and excess volatility. The competitive entry decision of rational financial institutions fails to internalize the negative information externality which expectational errors have on the exchange rate and therefore on the inference abilities of other traders. The improved risk sharing of a marginal trader may not compensate for the negative information externality of his trading errors. Market entry may therefore increase exchange rate volatility as documented by Ito, Lyons, and Melvin (1998). The competitive entry equilibrium in this situation is allocationally inefficient and implies excess volatility for the exchange rate. Taxation on financial institutions can decrease trader entry and volatility while increasing market efficiency.

The model has additional empirical implications. We show that expectational errors cannot account for the observed ratio of intramarket trading volume to total trading volume unless we allow for intermediary transactions in a decentralized market structure. The model can explain exchange rate heteroscedasticity and predicts a positive correlation between volatility and trading profits. Using data on the trading profits of U.S. banks, we do find evidence for a positive correlation.

Appendix A: The Filtration Problem

Proof of Proposition 1. We need to solve a standard linear filtration problem.[26] The state of the market process is denoted $z_t = (\Theta_t, \Psi_t)^T$ and the observable variables by dR_t. Note that observing the asset price history P_t is identical to observing the excess return history $dR_t = dP_t - \overline{r} P_t dt$. For the vector $dw_t = (dw_\Theta, dw_\Psi)^T$ of Wiener processes we can write

$$\begin{aligned} dz_t &= a_{zz} z_t dt + b_z dw_t \\ dR_t &= (a_{R0} + a_{Rz} z_t) dt + b_R dw_t \end{aligned}$$

$$a_{zz} = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix} \qquad b_z = \begin{bmatrix} b_\Theta & 0 \\ 0 & b_\Psi \end{bmatrix}$$

$$a_{R0} = e_0 \qquad a_{Rz} = \begin{bmatrix} e_\Theta & e_\Psi \end{bmatrix} \qquad b_R = \begin{bmatrix} p_\Theta b_\Theta & p_\Psi b_\Psi \end{bmatrix},$$

where $e_0 = -p_0 \overline{r}$, $e_\Theta = -(a + \overline{r})\, p_\Theta$, and $e_\Psi = -(a + \overline{r}) p_\Psi$. Let the filtered process be $\widehat{z}_t = E(z_t \mid \mathcal{F}_t)$ and the conditional variance of the filters

[26] For a complete treatment of the filtration problem, see Liptser and Shiryayev (1977). Another application is Wang (1993).

be $o_t = E[(\widehat{z}_t - z_t)(\widehat{z}_t - z_t)^T \mid \mathcal{F}_t]$. The analysis concentrates on the steady state of the filtration process. It is assumed that the initial beliefs o_0 of the trader coincide with the asymptotic beliefs. Let the steady-state solution of the Riccati equation be denoted

$$o_\infty = \begin{bmatrix} o_{11} & o_{21} \\ o_{21} & o_{22} \end{bmatrix}.$$

The steady-state solution o_∞ of the Riccati equation satisfies

$$0 = a_{zz} o_\infty + o_\infty a_{zz}^T + b_z b_z^T - h\left(b_R b_R^T\right) h^T, \tag{16}$$

where h is defined as

$$h = \begin{bmatrix} h_{\Theta R} \\ h_{\Psi R} \end{bmatrix} \equiv \left(o_\infty a_{Rz}^T + b_z b_R^T\right)\left(b_R b_R^T\right)^{-1}.$$

The filtered processes are characterized by

$$d\widehat{z}_t = \begin{bmatrix} d\widehat{\Theta}_t \\ d\widehat{\Psi}_t \end{bmatrix} = a_{zz}\widehat{z}_t dt + \begin{bmatrix} h_{\Theta R} \\ h_{\Psi R} \end{bmatrix} \left[b_R b_R^T\right]^{\frac{1}{2}} d\widehat{w}_t$$

$$d\widehat{w}_t = \left[b_R b_R^T\right]^{-\frac{1}{2}} \left[dR_t - e_0 dt - e_\Theta \widehat{\Theta}_t dt - e_\Psi \widehat{\Psi}_t dt\right]. \tag{17}$$

The observance of R_t implies that the processes Θ_t and Ψ_t and their respective filters $\widehat{\Theta}_t$ and $\widehat{\Psi}_t$ are constrained by

$$0 = e_\Theta(\widehat{\Theta}_t - \Theta_t) + e_\Psi(\widehat{\Psi}_t - \Psi_t). \tag{18}$$

Equation (18) allows us to simplify the Wiener process $d\widehat{w}$ given by Equation (17) to

$$d\widehat{w}_t = \left[b_R b_R^T\right]^{-\frac{1}{2}} b_R dw_t.$$

The restriction of Equation (18) in combination with the Riccati equation [Equation (16)] implies the following solution for the matrices o_∞ and h:

$$o_\infty = \begin{bmatrix} 1 & -\frac{p_\Theta}{p_\Psi} \\ -\frac{p_\Theta}{p_\Psi} & \left(\frac{p_\Theta}{p_\Psi}\right)^2 \end{bmatrix} \frac{p_\Psi^2 b_\Psi^2 b_\Theta^2}{2aV}$$

$$h = \begin{bmatrix} h_{\Theta R} \\ h_{\Psi R} \end{bmatrix} = \frac{1}{V} \begin{bmatrix} p_\Theta b_\Theta^2 \\ p_\Psi b_\Psi^2 \end{bmatrix}, \quad V \equiv p_\Theta^2 b_\Theta^2 + p_\Psi^2 b_\Psi^2.$$

The inference error process $\Delta_t = (\widehat{\Theta}_t - \Theta_t, \widehat{\Psi}_t - \Psi_t)$ satisfies

$$d\Delta_t = a_{zz}\Delta_t dt + b_\Delta dw_t,$$

where

$$b_\Delta = hb_R - b_z = \frac{b_\Theta b_\Psi}{V}\begin{bmatrix} -p_\Psi^2 b_\Psi & p_\Theta p_\Psi b_\Theta \\ p_\Theta p_\Psi b_\Psi & -p_\Theta^2 b_\Theta \end{bmatrix}. \qquad (19)$$

Appendix B: The Price Equilibrium

Proof of Propositions 2, 3, and 4. The asset demand of trader i is given by

$$\begin{aligned} X_t^i(p_0, p_\Theta, p_\Psi) &= \left[\left(\Psi+\Psi_t^i\right)dt + E^\mu(dR_t \mid \mathcal{F}_t)\right]\left[\rho E^\mu(dR_t^2 \mid \mathcal{F}_t)\right]^{-1} \\ &= \left[\Psi+\Psi_t^i + e_0 + e_\Theta(\Theta_t+\Delta_\Theta) + e_\Psi(\Psi_t+\Delta_\Psi)\right][\rho V]^{-1}, \end{aligned}$$

where $V \equiv p_\Theta^2 b_\Theta^2 + p_\Psi^2 b_\Psi^2$. The individual demand functions are aggregated to

$$\begin{aligned} X_t &= \int_{i\in\mathcal{T}} X_t^i(p_0, p_\Theta, p_\Psi)d\mu(i) \\ &= [e_0 + e_\Theta\Theta_t + (1+e_\Psi)\Psi_t + e_\Theta\Delta_\Theta + e_\Psi\Delta_\Psi]\left[\frac{\rho}{\lambda}V\right]^{-1}. \end{aligned}$$

The inference errors are (asymptotically) given by

$$\Delta_\Theta = \int_{s=t}^{\infty} e^{-a(s-t)}\,[b_{\Delta 11}dw_\Theta + b_{\Delta 12}dw_\Psi]$$

$$\Delta_\Theta = \int_{s=t}^{\infty} e^{-a(s-t)}\,[b_{\Delta 21}dw_\Theta + b_{\Delta 22}dw_\Psi]$$

and can be decomposed according to

$$e_\Theta\Delta_\Theta + e_\Psi\Delta_\Psi = k_\Theta\Theta_t + k_\Psi\Psi_t$$

for coefficients $k_\Theta \equiv (e_\Theta b_{\Delta 11} + e_\Psi b_{\Delta 21})/b_\Theta$ and $k_\Psi \equiv (e_\Theta b_{\Delta 12} + e_\Psi b_{\Delta 22})/b_\Psi$. Given the solution of Equation (19), we find $k_\Theta = k_\Psi = 0$. The market-clearing condition $X_t = \Theta_t + \gamma\left(P_t - \overline{P}\right)$ then implies three equations for the three equilibrium parameters:

$$e_0 = \frac{\rho}{\lambda}V\gamma(p_0 - \overline{P}) \qquad (20a)$$

$$e_\Theta = \frac{\rho}{\lambda}V(1+\gamma p_\Theta) \qquad (20b)$$

$$e_\Psi = -1 + \frac{\rho}{\lambda}V\gamma p_\Psi. \qquad (20c)$$

To determine the information content of the price, note that

$$\frac{p_\Theta}{p_\Psi} = -\frac{\rho}{\lambda}V.$$

The information content of the price follows as

$$\begin{aligned}\text{ICP} \;\equiv\; \frac{1}{o_{11}} &= \frac{2aV}{p_\Psi^2 b_\Psi^2 b_\Theta^2} = \frac{p_\Theta^2}{p_\Psi^2}\frac{2a}{b_\Psi^2} + \frac{2a}{b_\Theta^2} \\ &= \left(\frac{\rho}{\lambda}V\right)^2 \frac{1}{\text{var}(\Psi_t)} + \frac{1}{\text{var}(\Theta_t)},\end{aligned}$$

where $\text{var}(\Psi_t)$ and $\text{var}(\Theta_t)$ are the unconditional variances of the processes Ψ_t and Θ_t, respectively.

Appendix C: The Entry Equilibrium

Proof of Proposition 5. We can define a utility flow $F(V, \lambda)$ as

$$F(V, \lambda) \equiv \widehat{a}_\Pi - c(.) - \frac{1}{2}\rho\widehat{b_\Pi^2},$$

where the unconditional expectations

$$\widehat{a}_\Pi = E\left[a_\Pi(v^i)\right], \quad \widehat{b_\Pi^2} = E\left[b_\Pi(v^i) b_\Pi(v^i)^T\right]$$

are defined for $a_\Pi(v_t^i) = X(v_t^i)(e_\Theta \Theta_t + e_\Psi \Psi_t)$ and $b_\Pi(v_t^i) = X(v_t^i)(b_\Theta p, b_\Psi p_\Psi)$. The value function follows as

$$J^i(V, \lambda) = \max_{\{y(V,\lambda)\}} E\int_{t=0}^{\infty} e^{-rt}\left[d\Pi_t^i - \tfrac{1}{2}\rho\left(d\Pi_t^i\right)^2\right] = \frac{1}{r} y(V, \lambda) F(V, \lambda),$$

and the optimal hiring policy is

$$y(V, \lambda) = \begin{cases} 1 & \text{for} \quad F(V, \lambda) > 0 \\ 0 & \text{for} \quad F(V, \lambda) \le 0. \end{cases}$$

The financial institution is indifferent about market entry if $F(V, \lambda) = 0$. Note that for $\gamma = 0$, the asset demand simplifies to $X(v_t^i) = \frac{1}{\lambda}\Theta + \frac{1}{\rho V}\Psi^i$, and the market entry equilibrium is characterized by

$$F(V, \lambda) = \frac{1}{4a\lambda}\left(\frac{\rho}{\lambda} V b_\Theta^2 - \frac{1}{\frac{\rho}{\lambda}V} b_{\Psi^i}^2 - 4a\lambda c(.)\right) = 0.$$

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