

# Exchange Rates, Equity Prices, and Capital Flows

**Harald Hau**

Department of Finance, INSEAD

**Hélène Rey**

Department of Economics and Woodrow Wilson School,  
Princeton University

We develop an equilibrium model in which exchange rates, stock prices, and capital flows are jointly determined under incomplete foreign exchange (forex) risk trading. Incomplete hedging of forex risk, documented for U.S. global mutual funds, induces the following price and capital flow dynamics: Higher returns in the home equity market relative to the foreign equity market are associated with a home currency depreciation. Net equity flows into the foreign market are positively correlated with a foreign currency appreciation. The model predictions are strongly supported at daily, monthly, and quarterly frequencies for 17 OECD countries vis-à-vis the United States. Correlations are strongest after 1990 and for countries with higher equity market capitalization relative to GDP, suggesting that the observed exchange rate dynamics is indeed related to equity market development.

The last 25 years have been characterized by a remarkable increase in international capital mobility. While gross cross-border transactions in bond and equity for the United States were equivalent to only 4% of GDP in 1975, this share increased to 100% in the early 1990s and has grown to 245% by 2000. Furthermore, a growing proportion of these capital flows consists of equity as opposed to bank loans or government bonds<sup>1</sup> The

---

Deniz Igan provided outstanding research assistance. We thank Mike Woodford for his comments. Thanks also to participants in the 2004 American Econometric Association Winter Meetings, the 2003 European summer symposium in financial markets, the FX microstructure conference at the Stockholm Institute for Finance and at the London School of Economics (Financial Market Group), the 2002 NBER IFM summer institute, and in seminars at Columbia University, INSEAD, London Business School, ESCP-EAP (Paris), Georgetown University, George Washington University, and the IMF. We are both very grateful to the IMF Research Department for its warm hospitality and its stimulating environment while writing parts of this paper. Sergio Schmukler and Stijn Claessens provided the stock market capitalization data. This paper is part of a research network on "The Analysis of International Capital Markets: Understanding Europe's Role in the Global Economy," funded by the European Commission under the Research Training Network Program (Contract No. HPRNCTGE 1999E00067). Address correspondence to Harald Hau, Department of Finance, Boulevard de Constance, 77305 Fontainebleau Cedex, France, or e-mail: harald.hau@insead.edu.

<sup>1</sup> The London-based research firm Cross Border Capital reports that during the period 1975–1984, bank loans accounted on average for 39.5% of total outflows from major industrialized countries (60.3% of inflows), while equities accounted for only 9.5% of outflows (6.1% of inflows). During the 1985–1994 period, these proportions were reversed. Bank loans accounted only for 8.3% of outflows (16.3% of inflows), while equities jumped to 35.9% of outflows (31.6% of inflows). Similarly Lane and Milesi-Ferretti (2003) report that the ratio of foreign assets and liabilities to GDP has increased by 250% over the period 1984–2001. The same ratio computed only for equity and FDI has more than tripled over the same period. See also Frankel (1994) and Obstfeld and Taylor (2002).

increasing size and equity content of current capital flows has not yet inspired a new financial market paradigm for exchange rate theory, in which exchange rates, equity market returns, and capital flows are jointly determined.

Recently, positive exchange rate theory has advanced mostly outside the scope of traditional macroeconomic theory, plagued with its notoriously poor empirical performance [Meese and Rogoff (1982, 1983)] and with widespread pessimism about the explanatory power of macro variables in general to explain short- to medium-run exchange rate movements.<sup>2</sup> The empirical microstructure literature has examined the role of foreign exchange (forex) order flow defined as the difference between buy and sell orders. Evans and Lyons (2002a, 2002b, 2002c, 2003), Hau, Killeen, and Moore (2002), Killeen, Lyons, and Moore (2006), Lyons (2001), and Rime (2001) show that order flow from electronic trading systems has remarkably high correlation with contemporaneous exchange rate changes. Since order flow in the forex market is at least partly determined by investors' desires for portfolio shifts, these results suggest an important linkage between exchange rate dynamics and investor behavior. The most comprehensive order flow data are owned by global custodians like State Street, which undertake a large proportion of global equity clearing. Such (proprietary) data have been analyzed by Froot and Ramadorai (2005) and Froot, O'Connell, and Seasholes (2001). The results show that the impact of investor order flow on the exchange rate is very persistent and peaks at horizons of about a month for major currencies. Overall, the linkage between order flow and exchange rate movements appears very strong, notwithstanding the possibility that exchange rates may also move because of public information only and without any transaction. But the capital flow exchange rate linkage has not yet been imbedded in a theoretical framework in which order flow is derived from optimizing international investment behavior. There is still no model that bridges the gap between forex microstructure and macroeconomic fundamentals. To develop such a framework and explore its empirical implications are the main objectives of this article.

Our model departs from the existing literature on international portfolio choice in several important ways. First, we focus exclusively on modeling short- to medium-run exchange rate fluctuations and emphasize accordingly investment flows as the main determinant of the exchange rate. The flow determination can be understood as a consequence of limited forex arbitrage of risk averse speculators. Traditional portfolio balance models have typically imposed a purchasing power

---

<sup>2</sup> Frankel and Rose (1995) summarize the situation by saying that "[n]o model based on such standard fundamentals like money supplies, real income, interest rates, inflation rates, and current account balances will ever succeed in explaining or predicting a high percentage of the variation in the exchange rate, at least at short- or medium-term frequencies."

parity condition for the exchange rate determination. We abstract entirely from current account considerations, since the empirical literature has unambiguously shown that goods market variables do not explain exchange rate movements at the short- to medium-run frequencies.<sup>3</sup> Second, the existing stochastic portfolio balance models typically assume exogenous asset price processes. This precludes any feedback effects of optimal asset demands on the equilibrium price processes. Our framework fully endogenizes the equity market prices as well as the exchange rate and thereby allows a meaningful analysis of correlation structure of equity returns relative to exchange rate returns and equity flows. Third, unlike this literature, we make the assumption that bond supplies are infinitely elastic, which allows us to significantly simplify the dynamics of bond holdings in our model. Fourth, our most important structural assumption concerns incomplete forex risk trading. In complete markets and in a completely symmetric two-country model with equal market capitalizations, exchange rate risk hedging is a free lunch [Karolyi and Stulz (2002) and Perold and Schulman (1988)]. Investors in the home country can simply swap and eliminate forex risk by trading it with foreign investors holding the reciprocal risk. Under full forex risk hedging, the domestic and international investment problems are alike, unless we separate home and foreign investors by asymmetric information, different consumption opportunities, or preferences. But the evidence on forex hedging strongly suggests that market completeness represents a highly counterfactual benchmark. We have survey evidence on mutual funds and other institutional investors, which manage a large proportion of U.S. foreign equity investments. Their lower transaction costs and higher financial sophistication make them better candidates for forex risk trading compared to individual investors. Do they swap forex risk with their foreign counterparts? Levich, Hayt, and Ripston (1999) surveyed 298 U.S. institutional investors and found that more than 20% were not even permitted to hold derivative contracts in their investment portfolio. A further 25% of institutional investors were formally unconstrained, but did not trade in derivatives. The remaining 55% of institutional investors hedged only a minor proportion of their forex exposure. For the full sample, Levich, Hayt, and Ripston (1999) calculated that forex risk hedging concerned only 8% of the total foreign equity investment.<sup>4</sup> Portfolio managers cited

<sup>3</sup> Hence, we make the polar assumption of the one chosen by the New Open Economy Macroeconomics literature, which tends to focus exclusively on the goods market and to shut down the capital account by assuming specific utility functions [see for example Obstfeld and Rogoff (2000)]. Note that although this literature is unsuccessful at explaining exchange rate movements, it is well suited to address a host of other important issues that our simple model cannot tackle. We therefore see our approach and the existing literature as complementary.

<sup>4</sup> We also consulted market experts in two large U.S. custodians. Independent sources at both State Street Bank and Citibank estimated the notional forex hedge at <10%. This confirms the survey evidence.

monitoring problems, lack of knowledge, and public and regulatory perceptions as most important reasons for the restricted forex derivative use. The development of the derivative market notwithstanding, only a minor proportion of the total macroeconomic forex return risk seems to be separately traded and eliminated. The typical foreign equity investor holds currency return and foreign equity return risk as a bundle.

Exposure to exchange rate risk implies that the international investor generally cares about both the volatility of the exchange rate and the correlation structure of exchange rates and foreign equity returns. For example, higher exchange rate volatility tends to induce a home equity bias. On the other hand, a negative correlation between forex rate returns and foreign stock market returns reduces the return volatility in home currency terms and makes foreign investment more attractive. Portfolio choice therefore depends on exchange rate dynamics. But dynamic portfolio choice should simultaneously affect the exchange rate. Differences in the performance or value of the home and foreign stock market change the exchange rate risk exposure of both investors and give rise to portfolio rebalancing. Such dynamic rebalancing of equity portfolios then initiates forex order flow, which in turn induces exchange rate movements.

We capture this interaction between optimal portfolio choice under market incompleteness and exchange rate dynamics in a simple model. In particular, exchange rates, portfolio equity flows, and equity returns are jointly and endogenously determined. But to preserve tractability and gain some intuition on how the interaction between exchange rate and equity markets operate, we had to make some strong assumptions. We assume that in each of the two countries of our world economy, there is a constant risk-free interest rate and an exogenous stochastic dividend process for the equity market. Domestic and foreign investors are risk averse and maximize a simple trade-off between instantaneous trading profits and their variance. Any portfolio shift or dividend repatriation triggers an increase in demand for forex as investors do not hold foreign currency balances and all asset transactions are undertaken in the currency of denomination of the asset. Hence, a net outflow out of the domestic economy (which, for example, stems from purchases of foreign equities by domestic residents) translates into purchases of the foreign currency. Conversely, a net capital inflow leads to purchases of the domestic currency.<sup>5</sup>

One important building block of our model is the forex market. The exchange rate is determined under market clearing in the forex market where private investor order flows that stems from portfolio rebalancing

---

<sup>5</sup> Since equity flows account for above 30% of gross capital flows for developed countries in the early 1990s, the customer order flow they generate is quantitatively important. We also note that the gross cross-border equity volumes are roughly of a similar magnitude as the customer-dealer segment of the FX spot market.

and dividend repatriation meet a less than fully elastic forex supply of liquidity-providing financial institutions. This implies that order flow drives the exchange rate in accordance with the empirical findings in the recent finance microstructure literature.

The model we develop has testable implications regarding the relative volatilities of equity and exchange rate returns; correlations between stock index (excess) returns and exchange rate returns; and correlations between portfolio flows and exchange rate returns. We highlight here the three main empirical implications of our model:

- Market incompleteness in combination with a low price elasticity of forex liquidity supply generates exchange rates which are almost as volatile as equity prices.
- Higher returns in the home equity market (in local currency) relative to the foreign equity market are associated with a home currency depreciation: the model predicts what amounts to an “uncovered equity parity” condition.
- Net equity flows into the foreign market are positively correlated with a foreign currency appreciation.

We confront these model predictions with the data. Ratios of exchange rate volatility to equity return volatility are generally smaller than one and in the range replicated by the model. Return correlations are examined with daily, monthly, and quarterly stock index and exchange rate return data for 17 OECD countries. Strong statistical evidence is produced for a negative correlation between excess returns on foreign over U.S. equity and returns on the foreign currency as predicted by the model. Hence, both theory and evidence contradict the conventional wisdom that a strong equity market comes with a strong currency. We also highlight that these findings are produced at high statistical significance in contrast to the well-known failure of uncovered interest parity for the same set of countries. The evidence for the negative correlation between excess equity returns and exchange rate is strongest for the post-1990 period, when equity markets became more open. Cross-sectionally, we find that the negative correlation is more pronounced for countries with the most developed equity markets. Finally, we also use monthly equity flow data on the same OECD countries to verify the portfolio flow implications. In accordance with the model, the pooled regressions reveal a positive correlation between equity flows into the foreign market and the appreciation of the foreign currency.

The main intuition behind the “uncovered equity parity” condition is one of portfolio rebalancing. Whenever foreign equity holdings outperform domestic holdings, domestic investors are exposed to higher relative exchange rate exposure. They repatriate some of the foreign equity wealth to decrease the exchange rate risk. By doing so, they sell the foreign

currency, and this leads to a foreign currency depreciation. Therefore, portfolio rebalancing creates a negative correlation between equity market return differentials and exchange rate return.

Section 1 discusses the literature before we describe the model in section 3. In section 4, we solve the model for two special cases, namely the case of financial autarky and full integration in a complete market setting. These two polar cases provide two benchmarks for the general case of financial integration under market incompleteness explored in section 5. We summarize the most important testable implications in section 6 before confronting them with the data in section 7. Conclusions follow in section 8.

## 1. The Setting

It is useful to situate our analysis in the existing exchange rate literature. Our approach differs from previous studies in the following respects: (i) the emphasis on equity flows relative to the new open macroeconomics literature, (ii) the microfoundations for the asset demands and the endogeneity of the asset price processes relative to traditional portfolio balance models, (iii) the financial market incompleteness assumption relative to the real business cycle literature, (iv) the endogeneity of the order flows relative to the forex microstructure literature, and (v) the explicit modeling of the exchange rate relative to the finance literature.

Macroeconomic theory has recently emphasized better microfoundations together with a more rigorous modeling of the dynamic current account. This approach is exemplified by Obstfeld and Rogoff (1995) and surveyed in Lane (2001). But international equity markets do not play an important role in this framework. While monopolistic profits occur in these models, they typically accrue entirely to domestic residents and therefore do not give rise to any equity flows. In the spirit of the traditional asset market approach to exchange rates [surveyed by Branson and Henderson (1985)], we view short-run exchange rate movements determined by financial market. We extend existing portfolio balance models by allowing microfoundations of the asset demands and an endogenous determination of the equity price and exchange rate processes under market clearing. We obtain sharper testable implications for the correlation structure of forex returns, equity returns, and equity flows.

Our analysis features incomplete forex risk trading as an important structural assumption. To the extent that real business cycle models allow for international asset trade, they typically examine the resulting exchange rate dynamics in a complete market setting.<sup>6</sup> In this idealized

---

<sup>6</sup> Capital market incompleteness set our model apart from the Lucas (1982) model and much of the stochastic dynamic general equilibrium literature.

setting, all benefits from international exchange rate risk trading are realized. We argue that this assumption is at odds with current evidence on very low hedge ratios for foreign equity investment as discussed above. In our view, the market trades equity fairly frictionlessly across borders, but fails to realize the full benefit of trading the associated forex risk. This market incompleteness is not related to the absence of the markets (forex derivatives exist), but rather to transaction and agency costs of using them.

This article is inspired by the new empirical literature on the microstructure of the forex market. Order flow is identified as an important determinant of exchange rate dynamics. We interpret this literature as evidence for a less than infinitely-elastic forex supply and explore its consequences for optimal international portfolio investment.<sup>7</sup> In our model, forex order flow is derived endogenously from the optimal dynamic portfolio policy. Also, the time horizon for our analysis extends to several months unlike the high frequency focus in many microstructure models. These models also typically involve informational asymmetries, which play no role in our analysis.

Finally, our analysis relates to a recent literature on international equity flows. Some of this work is mainly empirical [Bekaert and Harvey (2000), Bekaert, Harvey, and Lumsdaine (2002), and Portes and Rey (2005); Richards (2005)]. Brennan and Cao (1997) and Griffin, Nardari, and Stulz (2002) also provide a theoretical analysis of foreign investment behavior. But both treated foreign investment like domestic investment by modeling only dollar returns. Instead of an exchange rate, home and foreign investors are separated by information asymmetries [Brennan and Cao (1997)] or by exogenous differences in return expectations [Griffin, Nardari, and Stulz (2002)]. Unlike these models, our framework assumes that foreign and home investors are separated by an exchange rate and pursue investment objectives in the currency of their respective residence.

## **2. The Model**

A world with two countries has a home and a foreign investor. Both investors are risk averse and can invest in risky home and foreign equities and in riskless bonds. Equities pay a continuous stochastic dividend flow. Purchase of a foreign equity by a domestic agent is settled in foreign currency and therefore requires a parallel purchase of foreign currency in the forex market. Increases in foreign equity holdings therefore generate

---

<sup>7</sup> In a similar spirit, Osler (1998) and Carlson and Osler (2000) model the exchange rate as the price equating supply and demand on a foreign exchange market where “current account traders” meet “rational currency speculators.”



an order flow in the forex market.<sup>8</sup> Investors do not hold money balances in equilibrium, since they are dominated by investments in the riskless bonds, which generate a nominal return of  $r$  and  $r^*$  in the respective local currencies. Central banks in both countries peg the interest rate so that the bond supplies are assumed to be infinitely price elastic. The two interest rates  $r$  and  $r^*$  are exogenous; for symmetry we impose  $r = r^*$ . Foreign dividend income is either reinvested in foreign equity or repatriated for home country investment. The supply of home and foreign equity is fixed and its price determined by market clearing. The supply of forex is less than perfectly elastic to reflect the empirical findings of the forex microstructure literature.

The evidence presented in the introduction strongly suggests that incomplete hedging of foreign investment is the more realistic benchmark compared to a world of full international exchange rate risk sharing. We therefore do not allow for short-selling of foreign bonds, as a short position in foreign bonds works as a forex hedge on the foreign equity investment. It is important to highlight that the short-sale constraint is binding in the steady-state equilibrium.<sup>9</sup> Intuitively, the home bond investment always strictly dominates the foreign bond investment under identical foreign and home bond returns and additional exchange rate risk on the foreign bond. Since home investors would like to hold a short position in foreign bonds to hedge the currency risk of their foreign equity position,<sup>10</sup> but are prohibited from doing so, they can at best choose a zero position of foreign bonds. To simplify the exposition and reduce notations, we therefore present the model as if investors were prevented from investing in foreign bonds altogether.<sup>11</sup>

The market structure is summarized as follows:

---

<sup>8</sup> We assume that when an agent purchases an equity from a foreign agent, she initiates the purchase of foreign exchange, so that our net currency flow coincides with the conventional definition of the order flow (net of buyer over seller initiated trades). Conversely if a domestic agent sells a foreign equity that she owns to a foreign investor, the sale is settled in foreign currency and the domestic agent then either converts the money in her own currency and buys a domestic bond or reinvest it in foreign assets. In the aggregate, this implies that there is a straightforward correspondence between net capital outflows out of the domestic economy and net domestic currency sale on one hand, and capital inflows into the domestic economy and net domestic currency purchases on the other hand.

<sup>9</sup> For a formal proof, see Appendix F of the working paper version [Hau and Rey (2003), NBER WP 9398].

<sup>10</sup> We study only equilibria in which investors have positive net foreign equity holdings in the steady state, since these are the only empirically relevant cases.

<sup>11</sup> Holding zero foreign bonds strictly dominates any positive foreign bond position only for steady-state values of the equilibrium price process and in some neighborhood around these values. Large deviations of the exchange rate from the long-run equilibrium may generate expectations of exchange rate changes which make positive foreign bond holdings optimal for one of the investors. In the case of large expected exchange rate changes, the equivalence of short-sale constraints and zero foreign bond holdings breaks down and we have to assume zero foreign bond holdings directly.



**Assumption 1: Asset Market Structure**

A home (*h*) and a foreign (*f*) stock market provide exogenous stochastic dividend flows  $D_t^h$  and  $D_t^f$  in local currency. Home and foreign investors can invest in both stock markets. In addition, each investor can invest in a domestic bond providing a riskless constant return  $r$  in the respective local currency.

Investors in our model are risk averse and their objective is to find an optimal trade-off between expected profit flow of their asset position and the instantaneous profit risk. Each investor measures profits in home currency. This investor objective implies particularly tractable asset demand functions which are linear in the ratio of expected return and instantaneous return risk.<sup>12</sup> Formally, we assume:

**Assumption 2: Investor Behavior**

Home and foreign investors are risk averse and maximize (in local currency terms) a mean–variance objective for the profit flow.<sup>13</sup> Home investors choose a portfolio of home and foreign equity,  $K_t = (K_t^h, K_t^f)$ , and foreign investors choose a portfolio of foreign and home equity,  $K_t^* = (K_t^{f*}, K_t^{h*})$ . Let  $dR_t = (dR_t^h, dR_t^f)^T$  and  $dR_t^* = (dR_t^{f*}, dR_t^{h*})^T$  denote the corresponding excess payoffs (in local currency terms over the local riskless bond) for domestic and foreign investors, respectively.<sup>14</sup> The corresponding stochastic excess profit flows follow as

$$d\Pi_t = K_t dR_t$$

$$d\Pi_t^* = K_t^* dR_t^*,$$

and the two investors optimize

$$\max_{\{K_t^h, K_t^f\}} \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} \left[ d\Pi_s - \frac{1}{2} \rho d\Pi_s^2 \right]$$

$$\max_{\{K_t^{f*}, K_t^{h*}\}} \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} \left[ d\Pi_s^* - \frac{1}{2} \rho d\Pi_s^{*2} \right],$$

<sup>12</sup> The common dynamic CARA utility framework produces similar asset demands which differ only by an additional intertemporal hedging demand component proportional to the covariance between asset excess return and the state variables. For reasons of tractability and simplicity, we chose not to model hedging demands to focus on the dynamic interactions between the equity and forex markets. The mean–variance objective here follows *inter alia* Hau (1998).

<sup>13</sup> For the time horizons relevant for our exercise (1 day to several months), prices can be considered to be sticky in local currency.

<sup>14</sup> The transposed vector is marked by  $T$ .

where  $\mathcal{E}_t$  denotes the rational expectation operator and  $\rho$  the investor risk aversion. Investor wealth evolves as

$$\begin{aligned} dW_t &= d\Pi_t + W_t r dt \\ dW_t^* &= d\Pi_t^* + W_t^* r dt. \end{aligned}$$

Both stock markets have to clear under the optimal asset demand. For simplicity, we normalize the quantity of outstanding equity to one. This implies

$$\begin{aligned} K_t^h + K_t^{h^*} &= 1 \\ K_t^f + K_t^{f^*} &= 1. \end{aligned} \tag{1}$$

as the two asset market-clearing conditions.

An additional market-clearing condition applies to the forex market with an exchange rate  $E_t$  expressed in units of foreign currency per units of domestic currency. Denoting home and foreign equity prices in local currencies by  $P_t^h$  and  $P_t^f$ , respectively, we can measure the equity-related capital flows  $dQ_t$  out of the home country (in foreign currency terms) as

$$dQ_t = E_t K_t^{h^*} D_t^h dt - K_t^f D_t^f dt + dK_t^f P_t^f - E_t dK_t^{h^*} P_t^h \tag{2}$$

The first two terms capture the outflow if all dividends are repatriated. But investors can also increase their holdings of foreign equity assets. The net purchases of foreign equity,  $dK_t^f$  and  $dK_t^{h^*}$  are captured by the third and fourth terms. Let us for example denote the euro area as the foreign and the United States as the home country. Then  $dQ_t$  represents the total net capital flow induced by equity trade (both dividend repatriation and net purchases) by U.S. investors into the euro area, in euro terms. An increase in  $E_t$  (denominated in euro per dollar) corresponds to a dollar appreciation against the euro. Any net capital flow out of a country is, in our model, identical to a net demand for foreign currency as all investment is assumed to occur in local currency. We can therefore also identify  $dQ_t$  with the equity trade-induced order flow for foreign currency in the forex market.<sup>15</sup> Furthermore, the above net capital flow out of the home country (or forex order flow) can be linearly approximated by

$$\begin{aligned} dQ_t^D &= (E_t - \bar{E}) \bar{K} \bar{D} dt + (\bar{E} K_t^{h^*} - K_t^f) \bar{D} dt + (\bar{E} D_t^h - D_t^f) \bar{K} dt \\ &\quad + (dK_t^f - \bar{E} dK_t^{h^*}) \bar{P}. \end{aligned} \tag{3}$$

<sup>15</sup> Remember that there is no trade in the foreign riskless bond in the steady-state equilibrium, so the forex order flow results only from equity trade and dividend repatriation.

where the upper bar variables denote the unconditional means of the stochastic variables. The linearization renders the analysis tractable.

The net forex order flow of investors is absorbed by liquidity-supplying banks which can buffer forex imbalances.<sup>16</sup> The following assumption characterizes the liquidity supply

**Assumption 3: Price-Elastic Excess Supply of Foreign Exchange**

*The foreign exchange market clears for a less than fully price-elastic excess supply curve with elasticity parameter  $\kappa$ . For an equilibrium exchange rate  $E_t$ , the excess supply of foreign exchange is given by*

$$Q_t^S = -\kappa(E_t - \bar{E})$$

where  $\bar{E}$  denotes the steady-state exchange rate level.

An increase in  $E_t$  (euro depreciation) decreases the excess supply of euro balances. Such an upward-sloping currency supply schedule can be interpreted literally as the set of limit orders in an electronic order book like Reuters Dealing 2000-2 or EBS.<sup>17</sup> Linearity of the supply curve simplifies the analysis. But we can also provide a more structural interpretation of the supply curve and the parameter  $\kappa$ . Such a microfoundation can either be based on imperfect intertemporal speculation of risk averse currency traders or alternatively be motivated by more long-run real trade effects.

Imperfect intertemporal speculation by risk averse currency traders may present the most plausible interpretation for less than fully elastic short-run excess supply curve. Currency speculators tend to sell dollars for euros when the dollar is high and buy dollars when the dollar is low. Different types of speculators provide liquidity at different horizons. Forex market makers take positions with half-lives measured in hours, while the half-lives of positions of proprietary trading desks, hedge funds, and non-financial corporations are measured in days, weeks, or months. Intertemporal risky arbitrage is imperfect since in practice traders do not have infinitely deep pockets. They face capital constraints [Shleifer and Vishny (1997)]. Generally, intertemporal speculation involves considerable risk and needs to be compensated by expected trading profit. Risk averse forex traders maximize a quadratic objective function strictly parallel to the equity investors (see assumption 2),

<sup>16</sup> A generalization of the model consists in allowing for additional current account imbalances given by  $CA_t dt = \gamma(\bar{E} - E_t) dt$ . Most microfounded trade or macroeconomic models generate a deficit for the current account of U.S. when the dollar is strong and vice versa ( $\gamma$  is the exchange rate elasticity of the current account). This generalization is straightforward and allows to reintroduce the goods market in the analysis.

<sup>17</sup> For a recent description of the microstructure of the foreign exchange market, see Hau, Killeen, and Moore (2002).

$$\max_{\{Q_t^S\}} \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} \left[ d\Pi_s - \frac{1}{2} \rho d\Pi_s^2 \right]$$

where currency trading profits are given by  $d\Pi_t = Q_t^S dE_t$  and  $\rho$  denotes the risk aversion of the currency speculators. The optimal liquidity supply  $Q_t^S$  is therefore given by  $Q_t^S = \mathcal{E}_t(dE_t) / \rho \sigma_e^2 dt$ . Assuming, as will be the case in our model, that the exchange rate follows an Ornstein–Uhlenbeck process reverting to a (constant) equilibrium value  $\bar{E}$  at speed  $\alpha_e$ , then the expected exchange rate change  $\mathcal{E}_t(dE_t)$  is equal to  $\alpha_e(\bar{E} - E_t) / \rho \sigma_e^2$ . Therefore, the optimal currency supply can be rewritten as  $Q_t^S = \alpha_e(\bar{E} - E_t) / \rho \sigma_e^2$  which is of the form  $-\kappa(E_t - \bar{E})$ . This highlights the role of rational exchange rate expectations for the currency traders despite the apparent absence of such expectations in the formulation of the above supply curve.<sup>18</sup> The parameter  $\kappa$  is determined by the degree of risk aversion of currency speculators and the statistical characteristics of the exchange rate process. In particular, if currency speculators are almost risk neutral, and therefore arbitrage aggressively deviations of the exchange rate from its long run level  $\bar{E}$ , then the supply of currency is very price elastic.<sup>19</sup>

Market clearing in the forex market requires  $Q_t^S = Q_t^D$ , and the forex rate is subject to the constraint

$$-\kappa dE_t = (E_t - \bar{E})\bar{K} \bar{D}dt + (\bar{E}K_t^{h*} - K_t^f)\bar{D}dt + (\bar{E}D_t^h - D_t^f)\bar{K}dt + (dK_t^f - \bar{E}dK_t^{h*})\bar{P}. \tag{4}$$

The exchange rate dynamics is therefore tied to the relative dividend flows,  $\bar{E}D_t^h - D_t^f$ , the relative level of foreign asset holdings  $\bar{E}K_t^{h*} - K_t^f$ , and their relative changes  $\bar{E}dK_t^{h*} - dK_t^f$ . The relative dividend flows are exogenous, but the optimal relative foreign equity holdings are endogenously determined and depend in turn on the exchange rate dynamics. We normalize  $\bar{E}$  to 1, because the two countries are symmetrical.

It is straightforward to express the excess payoffs (over the riskless asset) on a unit of home equity over the interval  $dt$  as  $dR_t^h$ . To characterize the foreign excess payoff  $dR_t^f$  in home currency, we use a linear approximation

<sup>18</sup> For a recent empirical analysis of intertemporal liquidity supply in the forex market, see Bjonnes, Rime, and Solheim (2005).

<sup>19</sup> An alternative microfoundation for the less than fully price-elastic currency supply resides simply in the dynamics of import and export markets for goods and services [see also Osler (1998)]. Most macroeconomic models incorporate short-run nominal price rigidities, and there are therefore competitiveness effects on international goods markets when a currency fluctuates. A country tends to run a trade surplus when the domestic currency is undervalued (relative to the long-run level  $\bar{E}$ ) and a trade deficit in the case of overvaluation. For example, a euro undervaluation ( $E_t > \bar{E}$ ) generates a euro area trade surplus and therefore an excess demand for euro balances. In this case, the parameter  $\kappa$  depends on the elasticity of substitution between domestic and foreign goods and the degree of nominal rigidity in the good markets.

around the steady-state exchange rate  $\bar{E} = 1$  and the steady-state price  $\bar{P}$ . Formally, excess payoffs are given as

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t \bar{P} + dP_t^f - dE_t dP_t^f - r[P_t^f - \bar{P}(E_t - 1)] dt \\ &\quad + [D_t^f - \bar{D}(E_t - 1)] dt \end{aligned}$$

for the home and foreign assets, respectively. Excess returns follow as  $dR_t^h/\bar{P}$  and  $dR_t^f/\bar{P}$ , respectively. The exchange rate component of the foreign payoff is given by  $-\bar{P}dE_t$  and the exchange rate return by  $-dE_t$ .<sup>20</sup>

Finally, we specify the stochastic structure of the state variables:

**Assumption 4: Stochastic Structure**

*The home and foreign dividends expressed in local currencies follow independent Ornstein–Uhlenbeck processes with identical variance and mean reversion  $\alpha_D > 0$  given by*

$$\begin{aligned} dD_t^h &= \alpha_D(\bar{D} - D_t^h)dt + \sigma_D dw_t^h \\ dD_t^f &= \alpha_D(\bar{D} - D_t^f)dt + \sigma_D dw_t^f. \end{aligned}$$

*The innovations  $dw_t^h$  and  $dw_t^f$  are independent.*

The mean reversion of all stochastic processes simplify the analysis considerably. We can now introduce variables  $F_t^h$  and  $F_t^f$  which denote the expected present value of the future discounted dividend flow,

$$\begin{aligned} F_t^h &= \mathcal{E}_t \int_{s=t}^{\infty} D_s^h e^{-r(s-t)} ds = f_0 + f_D D_t^h \\ F_t^f &= \mathcal{E}_t \int_{s=t}^{\infty} D_s^f e^{-r(s-t)} ds = f_0 + f_D D_t^f, \end{aligned}$$

with constant terms defined as  $f_D = 1/(\alpha_D + r)$  and  $f_0 = (r^{-1} - f_D)\bar{D}$ . The risk aversion of the investors and the market incompleteness with respect to forex risk trading imply that the asset price will generally differ from this fundamental value.

<sup>20</sup> The model is “closed” and there is no stock-flow inconsistency. A foreign equity purchase of the home investor is settled in foreign currency. But the foreign equity seller immediately reinvests this liquidity and holds zero money balances since money is a dominated store of value. He can either exchange it in the forex market if he reinvests in equity abroad, or bring it to his central bank at a fixed riskless rate. Central banks thus absorb the additional liquidity at the fixed rate  $r$ .

### 3. Two Special Cases

It is instructive to explore two special variations of our model. First, we cover the extreme case in which no foreign asset holdings are allowed. We refer to this case as financial autarky. It provides a useful closed economy benchmark for the stock market equilibrium, in which investors do not internationally share their domestic equity risk. The opposite extreme assumption is to allow both the equity risk and the exchange rate risk to be fully and separately traded. This second benchmark characterizes the international financial market equilibrium with complete risk sharing. Formally, it is identical to an economy with two freely tradeable assets. The exchange rate is a redundant price. As empirically most relevant, we consider a third case in which equity is freely traded but the exchange rate risk is not. We analyze this case in section 5.

Solving the model always requires three steps. First, we postulate a linear solution for the asset prices and the exchange rate. Second, we derive the optimal asset demand under the conjectured solution. Third, we impose the market-clearing conditions, show that the resulting price functions are indeed of the conjectured form, and finally solve for the coefficients. To provide for a more coherent exposition, we summarize our results in various propositions. All derivations are relegated to appendices.

#### 3.1 Financial autarky

Under financial autarky, the home investor's foreign equity position ( $K_t^f$ ) and the foreign investor's home equity position ( $K_t^{h*}$ ) are assumed to be zero. All domestic assets are owned by domestic investors, hence

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

The financial market equilibrium for the home and foreign equity market can be determined separately. Proposition 1 states the result:

#### **Proposition 1: Equilibrium under Financial Autarky**

*Assume a two-country world in which home investors hold the domestic asset and foreign investors the foreign asset. The home and foreign stock market prices are given by*

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \end{aligned}$$

*with  $p_0 = -\rho\sigma_R^2/r$  and  $p_F = 1$ . The (instantaneous) return volatility follows as  $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$ .*

**Proof:** See Appendix A.

A price parameter  $p_F = 1$  implies that the asset prices are proportional to their fundamental values  $F_t^h$  and  $F_t^f$ , respectively. The fundamental values represent the expected discounted future cash flows. The risk aversion of the investors is reflected in the coefficient  $p_0 < 0$ , which captures the equity risk premium as a price discount. It is proportional to the investor risk aversion  $\rho$  and the instantaneous variance  $\sigma_R^2$  of the excess return processes. These equilibrium results are standard for a closed economy with a fixed asset supply and mean–variance preferences for the investor.

### 3.2 Equilibrium with complete risk sharing

A second model variation consists in the full risk-sharing benchmark. Forex risk can then be fully traded either through derivative contracts or through short sales of the foreign riskless bond. Perfect and complete risk trading results in the elimination of all exchange rate risk. Intuitively, home and foreign investors hold exactly opposite and off-setting exchange rate risk in their global equity portfolio. They just need to swap the forex risk and thereby eliminate it.<sup>21</sup> The resulting financial market equilibrium is stated in proposition 2:

#### Proposition 2: Equilibrium with Complete Risk Sharing

*The home and foreign stock market prices and the exchange rate are given by*

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1 \end{aligned}$$

where we define  $p_0 = -\rho\sigma_R^2/2r$ , and  $p_F = 1$ . The (instantaneous) return volatility follows as  $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$ . The domestic and foreign portfolio positions of the two investors are equal and constant with

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f^*} & K_t^{h^*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

**Proof:** An identical riskless rate in the home and foreign country under complete markets implies a constant exchange rate,  $E_t = 1$ . The complete solution is derived in Appendix B.

First, we note that the exchange rate is constant. In a world of perfect risk sharing, the two-country model is not different from one domestic

<sup>21</sup> The exchange rate risk can be fully eliminated because of the symmetry of the two-country setup. In a more general model, forex risk can only be partially eliminated through swapping.



economy with two stocks. Home and foreign investors each hold equal and constant shares of the world market portfolio. The asset prices move one to one with their fundamental values,  $F_t^h$  and  $F_t^f$ , respectively. The risk sharing across the two investor groups implies that the asset price risk discount  $p_0 < 0$  is only half as large as in the autarky case for the same return volatility  $\sigma_R^2$ . This implies lower average asset returns under market integration. Evidence that financial integration indeed reduces market stock returns is provided by Bekaert and Harvey (2000), Henry (2000), and Stulz (1999) among others. These authors show reduced capital costs or excess returns on equity for emerging countries following their capital market liberalization.

We further highlight that complete forex risk trading implies no particular correlation structure between exchange rate and equity returns. The exchange rate is a redundant price and constant. This implication is of course at odds with the high exchange rate volatility observed in practice. But it provides a useful benchmark for the following section which explores the case of equity market integration under incomplete exchange rate risk trading.

#### **4. Foreign Investment under Incomplete Risk Sharing**

We now treat the case in which a forex market allows investment in the foreign equity, but exchange rate risk trading is incomplete. If the exchange rate moves stochastically, home investors with foreign equity holdings incur an additional exchange rate risk in addition to the risk of the stochastic dividend flow. Foreign investors hold the opposite risk due to ownership stakes in foreign equity. If this reciprocal exchange rate risk were tradeable, it could be perfectly eliminated as assumed in the perfect market case discussed in section 4.2. But now we assume that such forex risk trading does not occur.

The non-tradeability of the forex risk not only excludes derivative contracts, but also requires that investors cannot short-sell the foreign riskless asset. Short-selling of foreign riskless assets effectively amounts to a separate trading of the exchange rate risk. As discussed before, assuming a no short-sale constraint on the riskless foreign asset implies zero foreign bond holdings in the steady state. If unconstrained, investors should seek a short position in the foreign riskless asset equivalent to their foreign equity stake. But they would not seek a long position which adds exchange rate risk to the portfolio. The short-selling constraint is binding. Setting the foreign bond position to zero does not represent an additional restriction.

##### **4.1 Exchange rate dynamics**

Before we conjecture the exchange rate dynamics under incomplete markets, it is useful to highlight two principal equilibrium forces which shape

this dynamics. The first equilibrium tendency is governed by the less than fully elastic supply for forex order flow. Forex order flow  $dQ_t^D$  in Equation (3) is accommodated by financial institutions which finance these home outflows according to an upward-sloping supply curve. The elasticity of forex liquidity supply certainly influences the impact of net order flow on the exchange rate and indirectly the adjustment speed towards the steady-state exchange rate,  $\bar{E}$ . We associate the supply-induced mean reversion with a first characteristic root (labeled  $z$ ). A second important parameter for exchange rate dynamics is the mean reversion of the dividend processes. This mean reversion  $\alpha_D$  is exogenous, and any feedback effect from the exchange rate dynamics to the dividend process is ruled out by assumption.

An important simplifying feature of our model is its symmetry between the home and foreign country. Symmetry implies that the exchange rate can depend only on differences between home and foreign country variables, but not on a country-specific variable itself. Otherwise the symmetry would be broken. The symmetry requirement also implies that exchange rate surprises can depend only on current and past relative dividend innovations,  $dw_s = dw_s^h - dw_s^f$ . These relative innovations are the only exogenous source of exchange rate dynamics.

Finally, we highlight the linearity of the model structure. The forex order flow constraint is linearized and the exogenous dividend dynamics is linear by assumption. Moreover, we have assumed a mean-variance utility function which translates linear dividend, price, and return processes into linear asset demands. It is therefore justified to restrict our attention to the class of linear exchange rate and price processes. The argument for two fundamental equilibrium forces explains why we focus on two state variables  $\Delta_t$  and  $\Lambda_t$ , both of which depend for reasons of model symmetry on current and past relative dividend innovations  $dw_s$  only.

The following proposition 3 states the conjectured exchange rate process and derives its implications for the order flow constraint (4).

**Proposition 3: Exchange Rate Dynamics**

*Assume that (i) equity prices  $P = (P_t^h, P_t^f)$  depend linearly on the exchange rate  $E_t$  and the dividend processes  $D_t = (D_t^h, D_t^f)$  and (ii) the exchange rate has the following linear representation*

$$E_t = 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t,$$

*with*

$$\Delta_t = D_t^h - D_t^f = \int_{-\infty}^t e^{-\alpha_D(t-s)} \sigma_D dw_s$$

$$\Lambda_t = \int_{-\infty}^t e^{z(t-s)} dw_s,$$

where  $z < 0$  and  $dw_s = dw_s^h - dw_s^f$ . Then it follows that the order flow constraint (4) is of the simple form

$$dE_t = k_1 \Delta_t dt + k_2 (E_t - 1) dt + k_3 dw_t,$$

where  $k_1, k_2,$  and  $k_3$  represent undetermined coefficients.

**Proof:** The derivation is provided in Appendix C1. We have to show that for a linear price and a linear exchange rate, investor utility maximization implies optimal foreign equity demands  $K_t^{h*}, K_t^f$  such that the expression  $(K_t^{h*} - K_t^f) \bar{D} dt + (dK_t^f - dK_t^{h*}) \bar{P}$  in Equation (4) is linear in  $E_t - 1, \Delta_t$  and  $dw_t$ .

Under linearity of the price and exchange rate processes, the order flow constraint simplifies to a differential equation in only two state variables  $\Delta_t$  and  $E_t - 1$ . This allows us to characterize the exchange rate dynamics as a system of two first-order differential equations,

$$\begin{pmatrix} d\Delta_t \\ dE_t \end{pmatrix} = \begin{pmatrix} -\alpha_D & 0 \\ k_1 & k_2 \end{pmatrix} \begin{pmatrix} \Delta_t \\ E_t - 1 \end{pmatrix} dt + \begin{pmatrix} \sigma_D \\ k_3 \end{pmatrix} dw_t.$$

The associated characteristic roots are  $-\alpha_D$  and  $k_2$ . A stable solution requires  $k_2 < 0$ . The exchange rate solution can then be written as a linear combination  $e_{\Delta} \Delta_t + e_{\Lambda} \Lambda_t$  of the two eigenvectors

$$\Delta_t = \int_{-\infty}^t e^{-\alpha_D(t-s)} \sigma_D dw_s$$

and

$$\Lambda_t = \int_{-\infty}^t e^{k_2(t-s)} dw_s,$$

as conjectured in proposition 3.

To find the solution parameters, we have to impose the market-clearing conditions (1) and determine the steady-state levels for the equity price,  $\bar{P}$ , and the foreign equity holding,  $\bar{K}$ . Non-negative (steady state) prices ( $\bar{P} > 0$ ) and positive (steady state) home and foreign equity holdings

( $0 < \bar{K} < 1$ ) imply further restrictions on the parameter domain of our model. In particular we have to impose an upper bound  $\bar{\rho}$  on the risk aversion and a lower bound  $\bar{\kappa}$  on the elasticity of the forex liquidity supply to obtain plausible steady-state values.

Proposition 4 characterizes the equilibrium properties:

**Proposition 4: Existence and Uniqueness of the Incomplete Risk-Sharing Equilibrium**

*Let the economy be characterized by assumptions 1 to 4. For a sufficiently low risk aversion of the investors  $\rho < \bar{\rho}$  and a sufficiently price-elastic forex supply  $\kappa > \bar{\kappa}$ , there exists a unique stable linear equilibrium*

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h + p_\Delta \Delta_t + p_\Lambda \Lambda_t \\ P_t^f &= p_0 + p_F F_t^f - p_\Delta \Delta_t - p_\Lambda \Lambda_t \\ E_t &= 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t, \end{aligned}$$

where we define  $F_t^h$  and  $F_t^f$  as the expected present values of the future home and foreign dividend flows, respectively (as in section 3). The variable  $\Delta_t = D_t^h - D_t^f$  represents the relative dividend flows for the two countries and  $\Lambda_t$  a weighted average of past relative dividend innovations decaying at an endogenous rate  $z < 0$  as defined in proposition 3. The price parameters can be signed as

$$p_0 < 0, p_F = 1, p_\Delta > 0, e_\Delta < 0, e_\Delta \sigma_D + e_\Lambda < 0$$

Optimal portfolio holdings are given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{h*} & K_t^{f*} \end{pmatrix} = \begin{pmatrix} 1 - \bar{K} & \bar{K} \\ 1 - \bar{K} & \bar{K} \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \frac{1}{2\rho} (m_\Delta \Delta_t + m_\Lambda \Lambda_t)$$

for the parameters  $m_\Delta < 0$ , and  $m_\Lambda > 0$  defined in Appendix C1.

**Proof:** For a derivation see Appendix C.

As in the previous full risk-sharing case, we find that investor risk aversion requires an equity risk premium in the form of a price discount  $p_0 < 0$ . As before, a coefficient  $p_F = 1$  implies that the equity price reflects the fundamental value of expected future dividends,  $F_t^h$  and  $F_t^f$ , respectively. Moreover, two new stochastic terms  $\Delta_t$  and  $\Lambda_t$  influence asset prices and the exchange rate. These additional terms reflect changes in the asset prices and exchange rate dynamics induced by the incompleteness of forex risk trading. The exchange rate is no longer constant and exchange rate volatility imply asymmetric holdings of home and foreign equity. In addition, the optimal portfolio positions change proportionally to  $m_\Delta \Delta_t + m_\Lambda \Lambda_t$ . The dynamic equilibrium is characterized by constant

rebalancing of the optimal portfolios. We therefore have endogenous equity purchases and sales as a result of optimal equity risk trading under constrained forex risk trading. The net equity flows and the corresponding forex order flow in turn generate the equilibrium exchange rate dynamics under the price elastic forex liquidity supply.

#### 4.2 Economic interpretation

Investors in the two countries care about nominal trading profits in their domestic currency. This does not imply, however, that they only invest in home assets. Given that foreign asset investment provides an equity risk diversification benefit, foreign equity ownership is desirable for the home investor. But this diversification benefit comes at the cost of exchange rate risk on foreign equity positions and foreign dividend income. Incomplete international exchange rate risk trading implies that this exchange rate risk is not eliminated.

Moreover, the respective exposure of the home and foreign investor to forex risk varies with the relative performance of the home and foreign equity market. Any outperformance of the foreign equity market over the home equity market will tend to increase the relative exposure of the home investor to exchange rate risk and therefore alters his trade-off between diversification benefits and the costs of bearing currency risk. The foreign investor on the other hand does not face any currency risk on the higher market capitalization of the foreign market which provides payoffs in his own currency. The foreign investor therefore has a comparative advantage for holding a larger share of the foreign market capitalization given its relative value increase. The home investor can be expected to decrease his foreign position and his forex exposure. We refer to this as the risk rebalancing channel for capital flows. The risk rebalancing channel implies an optimal investment behavior for the home investor which is countercyclical to the foreign market excess performance. Inspection of proposition 4 shows that for high foreign market fundamentals ( $F_t^h - F_t^f < 0, \Leftrightarrow \Delta_t = D_t^h - D_t^f < 0$ ), the foreign holdings of the home investors ( $K_t^f$ ) decreases by the term  $-\frac{1}{2\rho}m_\Delta\Delta_t < 0$ , where  $m_\Delta < 0$ . This portfolio rebalancing constitutes an optimal adjustment of his equity positions in the face of exchange rate risk and incomplete forex risk trading.<sup>22</sup>

But a second effect creates additional capital flows. A higher relative foreign market capitalization coincides with relatively higher foreign dividend flows ( $\Delta_t dt = (D_t^h - D_t^f)dt < 0$ ). In the symmetric steady state, in which home and foreign investors hold equal shares of equity abroad, the home investor's foreign dividend income will exceed foreign investor's

<sup>22</sup> For developed equity markets, trading costs like fees and settlement costs are sufficiently small to be unlikely to impede portfolio adjustment. As for the price impact of trades, it is fully incorporated into the model since investors rationally foresee the equilibrium price impact of their reallocations.

dividend income from home country investments. Reciprocal dividend repatriation therefore creates an additional foreign capital outflow. We refer to this as the dividend repatriation channel.

A new aspect of our analysis is to consider the endogenous exchange rate reaction to both the dividend repatriation channel and the risk rebalancing channel. Both channels create a foreign capital outflow (for  $\Delta_t < 0$ ). The less than fully elastic currency supply implies that the foreign capital outflow generates an excess demand for home (dollar) currency and its appreciation. In other words, the foreign equity market excess returns come with a foreign currency (euro) depreciation. This explains why the coefficients  $p_\Delta > 0$  and  $e_\Delta < 0$  in proposition 4 have opposite signs. We can formally summarize this effect as follows:

**Corollary 1: Negative Correlation of Foreign Stock and Forex Returns**

*Under incomplete forex risk trading, foreign stock returns ( $dR_t^f/\bar{P}$ ) and exchange rate returns ( $-dE_t$ ) are negatively correlated, hence*

$$\mathcal{E}_t(-dE_t dR_t^f/\bar{P})dt = \mathcal{E}_t(dE_t dR_t^h/\bar{P})dt < 0.$$

**Proof:** Appendix E.

The negative correlation implies that the exchange rate provides a partial but automatic hedge against foreign equity risk. When foreign stock market returns are high, the foreign currency depreciates and vice versa. This reduces the return risk of foreign investment in home currency terms and increases the (steady-state) demand for foreign equity.

We also highlight the role of the risk rebalancing channel (and therefore incomplete forex risk trading) for magnifying exchange rate changes. If the exchange rate adjustment just counterbalanced high foreign dividend outflows ( $\Delta_t > 0$ ) without additional portfolio rebalancing, the flow constraint (4) would only consist of the terms  $-\kappa dE_t < 0$ ,  $(E_t - \bar{E})\bar{K}Ddt < 0$  and  $(D_t^h - D_t^f)\bar{K}dt > 0$ . But investors adjust their optimal portfolio holdings to the exchange rate dynamics, and these equilibrium portfolio shifts influence the exchange rate change through the additional terms  $(K_t^{h*} - K_t^f)\bar{D}dt$  and  $(dK_t^f - dK_t^{h*})\bar{P}dt$  in Equation (4). A low home country exchange rate ( $E_t$  low) makes foreign equity holdings relatively more attractive for the home investor since the value of foreign dividends in domestic currency is high, implying  $K_t^{h*} - K_t^f < 0$  (or  $m_\Delta < 0$ ); an expected appreciation leads to a expected net equity flow into the domestic market  $\mathcal{E}_t(dK_t^f - dK_t^{h*}) < 0$ . It follows from the flow constraint (4) that the endogenous portfolio shifts generally require a larger exchange rate appreciation  $-\kappa dE_t \ll 0$  than is needed to eliminate the imbalance in dividend income. In this sense, the exchange rate overshoots the dividend income

imbalance, and this overshooting is a consequence of incomplete forex risk trading.

Finally, we also note that imperfect intertemporal forex speculation is a necessary condition for these results. This can be verified by examining the limiting case of a completely price-elastic forex liquidity supply. In this case, the imperfect risk trading equilibrium converges to the special case of complete equity risk sharing:

**Proposition 5: Convergence to Complete Risk Sharing**

*The incomplete risk-sharing equilibrium (characterized in proposition 4) converges to the complete risk-sharing equilibrium (characterized in proposition 2) as the currency supply becomes infinitely price elastic, that is  $\kappa \rightarrow \infty$ .*

**Proof:** Appendix D.

In this limiting case, the investors can always exchange foreign dividend income at the constant exchange rate  $\bar{E} = 1$ . Optimal international equity risk sharing is achieved by equally shared ownership of the world equity portfolio. The infinitely elastic currency supply corresponds to a scenario of perfect intertemporal speculation in the forex market. If there existed a risk neutral forex trader with deep pockets, she could arbitrage away all exchange rate excess returns in which case the exchange rate would be constant and equal to one, as in the complete risk-sharing equilibrium. In practice, capital constraints for arbitraging speculators impose limits on the amount of intertemporal speculation [Shleifer and Vishny (1997)]. A relatively small supply elasticity of currency is therefore the correct benchmark.

In our model, exchange rate pegging would enable complete risk-sharing and be welfare-improving. But this result is certainly not robust since our setup merely outlines the benefits of a peg for equity market integration but has nothing to say about the effect of a fixed exchange rate on the real side of the economy. However, these real effects are likely to determine the feasibility and credibility of a fixed exchange rate regime.

**5. Model Implications**

We summarize the main empirical implications of our model, which concern the volatility of the exchange rate return relative to the equity return in section 6.1, the correlation structure of exchange rate and equity returns in section 6.2, and the correlation structure of exchange rate return and equity flows in section 6.3. We also discuss the effect of equity market development on the strength of our results in section 6.4.

**5.1 Exchange rate volatility**

Market completeness means that forex risk is widely and efficiently traded. Derivative trading or short-selling of bonds reallocate and largely



eliminate the forex risk of all international equity investors. Moreover, many market participants and their low aggregate risk aversion imply a very price-elastic forex supply. Home and foreign currency are then close substitutes. This limits the scope for forex order flow to generate considerable exchange rate volatility. Alternatively, if forex risk trading is restricted to a relatively small number of banks and hedge funds, then we expect a less price-elastic forex liquidity supply. In the latter case, forex order flow may result in considerable exchange rate movements.

Our model captures the elasticity of the forex supply in the parameter  $\kappa$ . Portfolio flows in the incomplete risk-sharing setting can generate considerable exchange rate volatility if  $\kappa$  becomes small. The volatility ratio of the exchange rate returns and the stock market returns (in local currency)

$$\sqrt{\frac{\text{var}(dE_t)}{\text{var}(dR_t^{f*}/\bar{P})}},$$

is decreasing in  $\kappa$ , when everything else is held constant. A high price elasticity of forex liquidity supply ( $\kappa$  large) implies a low forex volatility. Numerical simulations show that a decrease in the elasticity of the liquidity supply (lower  $\kappa$ ) comes with substantial forex volatility.<sup>23</sup> Specifically, when the elasticity of the liquidity supply is low, the volatility of the exchange rate returns amounts to 30–70% of the volatility of stock market returns, depending on the degree of risk aversion. If investor risk aversion  $\rho$  decreases, the volatility of the exchange rate increases since more risk averse traders engage less aggressively in arbitraging. We summarize this result as follows:

### **Implication 1: Exchange Rate Volatility**

*Market incompleteness in combination with a less than fully elastic forex liquidity supply can generate exchange rates which are almost as volatile as equity returns.*

## **5.2 Equity returns and exchange rate returns**

Market incompleteness implies a negative correlation structure between foreign equity returns and exchange rate returns as stated in Corollary 1. Because of the symmetry of the model, it is most convenient to state the correlation structure for differences of the foreign and home equity

<sup>23</sup> See the working paper version (NBER WP 9398 or CEPR DP 3735) for details. In the simulations, we keep constant the riskless rate  $r$  and the three parameters governing the dividend processes  $(\bar{D}, \alpha_D, \sigma_D)$ . We allow the degree of risk aversion  $\rho$  to take any value of the interval [0.04, 0.44]; the liquidity supply parameter can take any value of [20, 100].

returns in local currency, namely  $(dR_t^{f*} - dR_t^h)/\bar{P}$ . The following corollary provides the result:

**Corollary 2:**

*Under incomplete forex risk trading, a foreign currency appreciation and foreign excess returns (in local currency) over home market returns have a perfect negative correlation, hence*

$$\text{corr}\left[-dE_t, (dR_t^{f*} - dR_t^h)/\bar{P}\right] = -1.$$

**Proof:** See Appendix E.

For example, a U.S. equity market return shortfall relative to the European equity market  $((dR_t^{f*} - dR_t^h)/\bar{P} > 0)$  should *ceteris paribus* coincide with a dollar appreciation  $(dE_t > 0)$ . The negative correlation is perfect, because we have only two exogenous stochastic processes for the dividends which influence the model dynamics. For reasons of symmetry, return differences and exchange rate returns are driven exclusively by relative dividend innovations,  $dw_t = dw_t^h - dw_t^f$ . The instantaneous correlation between the local currency excess return can therefore only be either perfectly negative or positive or zero. Our analysis shows that the correlation is perfectly negative. Empirically, we cannot expect to find a perfectly negative correlation. Shocks other than dividend innovations and cross-country asymmetries will tend to reduce the absolute value of the correlation. As the empirically relevant implication, we therefore retain only the sign of the correlation:

**Implication 2: Differential Equity Returns and Foreign Exchange Rate Return**

*When foreign stock index returns in (local currency) are in excess of the U.S. stock index returns (in dollars), the foreign currency depreciates.*

To our knowledge, this particular correlation structure has not yet been related to financial structure in general and the incompleteness of forex risk trading in particular. We explore its empirical validity in section 7.2.

**5.3 Exchange rate returns and portfolio flows**

Exchange rates in our model are determined through a price-elastic response to forex order flow, which in turn originates partly in equity flows. It therefore seems appropriate to relate exchange rate returns directly to equity portfolio flows. Using the price equilibrium in proposition 4, it is straightforward to show that the equity flows into the foreign

market by home investors,  $dK_t^f = -dK_t^{f*}$ , and the equity flow into the home market by foreign investors,  $dK_t^{h*} = -dK_t^h$ , both correlate positively with the exchange rate return  $-dE_t$  and  $dE_t$ , respectively. Formally,

$$\mathcal{E}_t(-dE_t dK_t^f) = \mathcal{E}_t(dE_t dK_t^{h*}) = \frac{\kappa}{P} (e_\Delta \sigma_D + e_\Lambda)^2 dt > 0.$$

The symmetry of the model implies that the exchange rate return has the same absolute covariance with foreign net purchases of domestic equities and with domestic net purchases of foreign equities, but with opposite signs. We can express the net equity flow into the foreign country as the difference  $dK_t^f - dK_t^{h*}$ . This net flow exhibits a perfect positive correlation with the exchange rate return. Hence the following corollary:

**Corollary 3:**

*Under incomplete forex risk trading, a foreign currency appreciation and the net equity flow into the foreign country have a perfect positive correlation,*

$$\text{corr} \left[ -dE_t, \left( dK_t^f - dK_t^{h*} \right) \right] = 1.$$

**Proof:** See Appendix E.

Again, the correlation is perfect, because all variables for country differences or the exchange rate are governed by stochastic innovations which are proportional to the relative dividend innovations,  $dw_t = dw_t^h - dw_t^f$ . Country heterogeneity in other dimensions will certainly tend to decrease the correlation to a value below one. We therefore retain only the sign of the correlation as the empirically relevant model implication and refer to the U.S. as the home country:

**Implication 3: Forex Return and Net Equity Flows**

*A foreign currency appreciation is positively correlated with net equity flows into the foreign market.*

**5.4 The role of equity market development**

The correlation structure of equity and exchange rate returns was derived for integrated and frictionless equity markets. But equity market development and integration constitute a relatively recent phenomenon. Only in the 1990s did international equity trading become a prominent feature in international finance. Hence, we expect the empirical model implications to hold best for OECD country data over the last decade. We therefore examine the correlation structure separately over the entire data collection period and for two subsamples starting in 1990 and 1995. An increasingly negative correlation between foreign excess equity

returns and the forex rate return suggests that the correlation structure is indeed induced by increasing equity market integration.

Moreover, the evidence should be strongest for countries with relatively developed equity markets. Such equity market development can be crudely measured by the ratio of market capitalization to GDP. Alternatively, we can measure the integration of a local equity market into the world equity market by the ratio of gross equity trade to GDP. Both market development measures should be correlated with the magnitude of the predicted correlation structure. Such cross-sectional evidence suggests again that the exchange rate dynamics represents a financial market phenomenon.

We can summarize both the time-series and cross-sectional implications as follows:

**Implication 4: Negative Correlation and Equity Market Development**

*The magnitude of the negative correlation between foreign equity excess return and the exchange rate return should increase in the 1990s and should be strongest for countries with a high degree of equity market development as measured by the ratio of market capitalization to GDP or gross equity trade to GDP.*

**6. Evidence**

The empirical work focuses on OECD countries relative to the United States. OECD countries tend to have the most developed equity markets and are therefore most pertinent for the model. The United States represents by far the largest source and recipient of international equity flows. Furthermore, the most comprehensive bilateral asset flow data are available for the United States only. Within the OECD sample, we excluded three countries for which daily exchange rate data were not available over a sufficiently long time period: Iceland, Greece, and New Zealand. Belgium and Luxemburg are treated as one country because of their common currency. Canada was excluded because of its effective exchange rate fixing with the United States according to Reinhart and Rogoff (2004).<sup>24</sup> The remaining 17 OECD countries maintained flexible exchange rates relative to the United States and constitute our sample.

The daily equity index and exchange rate data are obtained from Datastream. We used the MSCI series for the end of the day stock index quote and the corresponding dollar exchange rates. Most daily price data are available since 1980. The data are screened for data outliers and errors and do not show any abnormal entries.

Portfolio flow data are more difficult to obtain. We use the so-called TIC data [Board of Governors of the Federal Reserve System (2004)].

<sup>24</sup> The same exchange rate consideration would also lead to the exclusion of Hong Kong and Singapore, which have developed equity markets but are not considered OECD countries.

Available on a monthly frequency since 1987, the TIC data record transactions in portfolio equities between U.S. residents and residents of foreign countries.<sup>25</sup> They allow us to compute net purchases of foreign equities by U.S. residents ( $dK_t^f$ ) and net purchases of U.S. equities by foreigners ( $dK_t^{h*}$ ). Cross-border equity flows have been growing sizably in the last decade.<sup>26</sup> Hence, we have to find a suitable normalization of the portfolio flow series. We consider a normalization for capital flows by market capitalization and alternatively by the average flows over the previous 12 months [as in Brennan and Cao (1997)]. Both methods produce very similar results and we only report tables with the normalization based on past average flows. The stock market capitalization data come from the S&P Emerging Markets Database.

### 6.1 Exchange rate volatility

First, we examine the volatility ratio of exchange rate returns to stock index returns. We calculate the standard deviation of the log returns of the dollar exchange rate and the stock index returns in local currency. Table 1 reports the ratio of the standard deviations for the entire data sample since 1980 [column (a)], the subsample since 1990 [column (b)], and the most recent period since 1995 [column (c)].

The volatility ratio over the full sample varies between 0.369 for Finland and 0.845 for Switzerland with a mean for all countries of 0.6215. Our theoretical framework can explain such high exchange rate volatility with a relatively low price elasticity of the forex liquidity supply. Comparing volatility ratios for the entire period since 1980 to the more recent subsamples since 1990 and 1995, we find declining volatility ratios for most countries. This can mostly be attributed to a decrease in exchange rate volatility. We can speculate that the elasticity of liquidity supply in the forex market (parameter  $\kappa$  in our model) might have increased over time. This would be consistent with increasing forex market depth in the more recent period.

### 6.2 Equity returns and exchange rate returns

The most important model implication concerns the negative correlation of equity return differential between foreign and home index returns expressed in the respective local currency and the exchange rate return. We calculate the return correlations based on daily returns for various data periods. Exchange rate returns are in foreign currency per dollar and

<sup>25</sup> For a thorough presentation of these data, see Warnock, Grier, and Lee (2001). We note that TIC data records transactions based on the residency of the seller and of the buyer. For example, a German equity sold in London by a U.S. resident to a U.K. Bank will be recorded as a sale of a foreign security by a U.S. resident to the U.K. In our model, this transaction will therefore be interpreted as a dollar pound transaction on the forex market. This inference can be flawed insofar as the real operation was actually performed in euro and not in Sterling or as it was realized on the behalf of a German equity trader.

<sup>26</sup> See Portes and Rey (2004) for a detailed study of the properties of these flows.

**Table 1**  
Volatility ratios of exchange rate and stock market index returns

$$\sqrt{\frac{\text{var}(dE_t)}{\text{var}(dR_t^f/P)}}$$

	(a) January 1, 1980– December 31, 2001	(b) January 1, 1990– December 31, 2001	(c) January 1, 1995– December 31, 2001
Australia	0.5850	0.6494	0.7070
Austria	0.8205	0.6272	0.6270
Belgium-Luxembourg	0.8386	0.8053	0.6770
Denmark	0.6951	0.6351	0.5540
Finland	0.3690	0.3388	0.2472
France	0.6081	0.5450	0.4785
Germany	0.6211	0.5537	0.4721
Ireland	0.5968	0.5921	0.5361
Italy	0.4901	0.4688	0.4198
Japan	0.6279	0.5444	0.5855
Netherlands	0.6555	0.6553	0.5196
Norway	0.4517	0.4937	0.5023
Portugal	0.6423	0.6530	0.5980
Spain	0.5920	0.5156	0.4478
Sweden	0.4664	0.4424	0.3766
Switzerland	0.8450	0.7241	0.6441
U.K.	0.6599	0.6037	0.4747
Mean	0.6215	0.5793	0.5216
SD	0.1328	0.1111	0.1159
Pooled data	0.4780	0.4754	0.4116

Reported are volatility ratios of daily (log) exchange rate returns and daily (log) foreign stock market index returns (in local currency) for various sample periods. The exchange rate  $E_t$  is expressed in foreign currency per dollar ( $dE_t > 0$  corresponds to a dollar appreciation), and the index  $f$  represents 1 of the 17 foreign OECD countries. The last row provides the result for the pooled data.

stock index returns are measured in local currency. The correlation evidence is produced at the daily, monthly, and quarterly frequency in Tables 2, 3, and 4, respectively.

Daily correlations in Table 2 provide strong statistical evidence in favor of our correlation hypothesis. The model prediction of a negative correlation is validated for most countries at a 1% statistical significance level.<sup>27</sup> Moreover, the correlations become more negative in the two more recent periods. The correlation in the pooled data decreases from  $-0.053$  over the entire period to  $-0.0761$  for the period since 1990 and to  $-0.0735$  for the most recent period since 1995. The correlation has grown more negative along with the equity market integration, which has intensified since the 1990s. The only countries for which the correlation is still positive after 1995 are Australia and Japan.<sup>28</sup> Overall, our evidence strongly supports

<sup>27</sup> Standards errors are corrected for heteroskedasticity and serial correlation.

<sup>28</sup> We conjecture that the Australian evidence might be tainted by the role of natural resource prices. Chen and Rogoff (2002) and Cashin, Cespedes and Sahay (2002) show indeed that the Australian exchange rate is strongly related to world commodity price fluctuations. They underline the specificity of this country in this respect. Japan on the other hand is special because international portfolio flows concern mostly bonds as opposed to equity.

**Table 2**  
Daily correlations of exchange rate and foreign stock market excess returns

	corr $\left[-dE_t, \left(dR_t^f - dR_t^h\right)/\bar{P}\right]$		
	(a) January 1, 1980– December 31, 2001	(b) January 1, 1990– December 31, 2001	(c) January 1, 1995– December 31, 2001
Australia	0.0558***	0.0304*	0.0242
Austria	-0.0186	-0.0291	-0.0201
Belgium-Luxembourg	-0.0438***	-0.0388**	-0.0226
Denmark	-0.0368***	-0.0495***	-0.0452*
Finland	-0.0954***	-0.1263***	-0.1847***
France	-0.1026***	-0.1638***	-0.1760***
Germany	-0.0805***	-0.1021***	-0.1448***
Ireland	-0.1003***	-0.0883***	-0.0739***
Italy	-0.0385***	-0.0355**	-0.0539**
Japan	0.0636***	0.0723***	0.0587**
Netherlands	-0.1674***	-0.2194***	-0.2052***
Norway	-0.0629***	-0.0956***	-0.0128
Portugal	-0.0253	-0.0339*	-0.0140
Spain	-0.0645***	-0.1301***	-0.1116***
Sweden	-0.0677***	-0.0510***	-0.0163
Switzerland	-0.1240***	-0.1632***	-0.1655***
U.K.	-0.0173	-0.1024***	-0.1042***
Mean	-0.0545	-0.0780	-0.0746
SD	0.0586	0.0728	0.0792
Pooled data	-0.0530***	-0.0761***	-0.0735***

Reported are correlations of daily (log) exchange rate returns,  $-dE_t$ , and the daily (log) foreign stock market index returns (in local currency) relative to the U.S. market index return (in dollars),  $(dR_t^f - dR_t^h)/\bar{P}$ , for various sample periods. The exchange rate  $E_t$  is expressed in foreign currency per dollar ( $dE_t > 0$  corresponds to a dollar appreciation). The index  $f$  represents 1 of 17 OECD countries and  $h$  the U.S. market. The model predicts  $\text{corr}[-dE_t, (dR_t^f - dR_t^h)/\bar{P}] < 0$ . We test whether the correlation is significantly different from zero using robust standard errors and denote by \*, \*\*, and \*\*\* significance at a 10, 5, and 1% level, respectively. The last row provides the result for the pooled data.

the predicted negative correlation. Regression evidence on the pooled data sample shows a strong negative correlation significant at the 1% level. A high level of statistical significance is also obtained if we aggregate countries in the European monetary system into single-country observation. The monthly return data in Table 3 provide very similar results. In the sample period since 1995, every OECD country features a negative correlation at the monthly frequency, and the average correlation in the pooled sample exceeds 20%. We consider this as economically highly significant. Again, we find that the correlation became more negative in the 1990s. For the entire data collection period from 1973 to 2002, the correlation is roughly half as strong as in the last decade.

Table 4 confirms our results on quarterly data for the period 1990 to 2002. We present regressions of exchange rate changes on return differentials for all the countries of our sample. The correlation is again negative and strongly significant for most countries. Furthermore, the variance of the exchange rate explained by our simple return differential variable is strikingly high for some countries. With a single variable,



**Table 3**  
**Monthly correlations of exchange rate and foreign stock market excess returns**

	$\text{corr} \left[ -dE_t, \left( dR_t^f - dR_t^h \right) / \bar{P} \right]$		
	(a) January 1980– December 2001	(b) January 1990– December 2001	(c) January 1995– December 2001
Australia	0.1796***	0.0102	-0.1415
Austria	-0.1020	-0.1998**	-0.1507
Belgium-Luxembourg	-0.2508***	-0.2569***	-0.1352
Denmark	-0.2179***	-0.2934***	-0.3358***
Finland	-0.1580**	-0.2570***	-0.1794**
France	-0.1230**	-0.3473***	-0.3118***
Germany	-0.1409**	-0.2871***	-0.3679***
Ireland	-0.2710***	-0.2805***	-0.2996***
Italy	-0.1308**	-0.1312	-0.1755**
Japan	0.6590	-0.0276	-0.2810***
Netherlands	-0.3403***	-0.3689***	-0.3059***
Norway	-0.0936	-0.1787**	-0.0264
Portugal	-0.0763	-0.1341*	-0.0669
Spain	-0.1250**	-0.2183***	-0.2090**
Sweden	-0.2287***	-0.2862***	-0.0930
Switzerland	-0.1761***	-0.2318***	-0.1376
U.K.	-0.1187*	-0.2778***	-0.2530***
Mean	-0.1009	-0.2169	-0.2041
SD	0.2248	0.1059	0.1012
Pooled data	-0.1232***	-0.2119***	-0.1901***

Reported are correlations of monthly (log) exchange rate returns,  $-dE_t$ , and the monthly (log) foreign stock market index returns (in local currency) relative to the U.S. market index return (in dollars),  $(dR_t^f - dR_t^h)/\bar{P}$ , for various sample periods. The exchange rate  $E_t$  is expressed in foreign currency per dollar ( $dE_t > 0$  corresponds to a dollar appreciation). The index  $f$  represents one of 17 foreign OECD countries and  $h$  the U.S. market. The model predicts  $\text{corr}[-dE_t, (dR_t^f - dR_t^h)/\bar{P}] < 0$ . We test if the correlation is significantly different from zero using robust standard errors and denote by \*, \*\*, and \*\*\* significance at a 10, 5, and 1% level, respectively. The last row provides the result for the pooled data.

namely equity return differential, we explain 30% of quarterly exchange rate movements in Spain, 28% in Sweden, and 25% in Germany. For the pooled data the  $R^2$  is 13%. These results offer a sharp contrast with the dismal performance of monetary variables in standard exchange rate models at quarterly horizons. The larger explanatory power of the local currency equity return differential for monthly and quarterly exchange rate changes compared to daily data may be explained by the sluggishness of portfolio rebalancing. Many investors may undertake rebalancing decisions not on daily, but rather at a weekly or monthly frequency.

The negative correlation has previously been noted by other researchers for some particular countries. But they were mostly puzzled by the lack of a coherent theoretical explanation. Brooks et al. (2001) for example document negative correlations between European equity excess returns over U.S. equity and the euro-dollar exchange rate. Interestingly, they discard their finding as “counter-intuitive” (p. 17), since it contradicts the popular view that a strengthening equity market should be mirrored by a

**Table 4**  
**Regressions of quarterly exchange rate on foreign stock market excess returns**

	Coefficients (January 1990–December 2001)		
	$\alpha$	$\beta$	Adjusted $R^2$
Australia	-0.0092 (0.0064)	-0.1124 (0.1336)	0.0215
Austria	-0.0113 (0.0078)	-0.2046** (0.0825)	0.1535
Belgium-Luxembourg	-0.0078 (0.0073)	-0.2613* (0.1430)	0.0659
Denmark	-0.0065 (0.0067)	-0.3266*** (0.0780)	0.2294
Finland	-0.0089 (0.0087)	-0.0734 (0.0451)	0.0340
France	-0.0068 (0.0065)	-0.3999*** (0.0783)	0.2447
Germany	-0.0080 (0.0068)	-0.3385*** (0.0656)	0.2467
Ireland	-0.0090 (0.0069)	-0.3372*** (0.0604)	0.2646
Italy	-0.0126 (0.0081)	-0.1423 (0.0972)	0.0462
Japan	-0.0004 (0.0119)	-0.0530 (0.1483)	0.0151
Netherlands	-0.0052 (0.0066)	-0.5273*** (0.1121)	0.2677
Norway	-0.0091 (0.0070)	-0.1646*** (0.0562)	0.0998
Portugal	-0.0116 (0.0069)	-0.1831** (0.0740)	0.1226
Spain	-0.0113 (0.0063)	-0.2847*** (0.0659)	0.3029
Sweden	-0.0095 (0.0082)	-0.2698*** (0.0969)	0.2809
Switzerland	-0.0009 (0.0083)	-0.3368** (0.1377)	0.1305
U.K.	-0.0045 (0.0073)	-0.2738** (0.1348)	0.0587
Mean	-0.0083 (0.0075)	-0.2523 (0.0947)	0.1477
SD	0.0030 (0.0014)	0.1237 (0.0339)	0.1096
Pooled data	-0.0081 (0.0057)	-0.2083*** (0.0426)	0.1261

Reported are regressions of quarterly (log) exchange rate returns on the quarterly (log) foreign stock market excess return (in local currency) relative to the U.S. stock market return (in dollars) for the period 1990–2002:

$$-dE_t = \alpha + \beta(dR_t^{f^*} - dR_t^h)/\bar{P} + \varepsilon_t.$$

The exchange rate  $E_t$  is expressed in foreign currency per dollar ( $dE_t > 0$  corresponds to a dollar appreciation). The index  $f$  represents 1 of 17 foreign OECD countries and  $h$  the U.S. market. The model predicts  $\beta < 0$ . We denote by \*, \*\*, and \*\*\* significance at a 10, 5, and 1% level, respectively. Newey–West-adjusted standard errors are reported in parentheses. The last row provides the result for the pooled data.

strengthening exchange rate.<sup>29</sup> Incomplete forex risk trading offers a coherent theoretical explanation for the observed correlation structure. From an empirical perspective, the negative correlation deserves to be highlighted because of its strong statistical significance and increasing magnitude. Moreover, it stands out relative to the empirical failure of uncovered interest parity for the same set of countries.<sup>30</sup>

One could theoretically offer an alternative hypothesis for this negative correlation. A depreciation of the exchange rate could be associated with higher equity returns via a competitiveness effect for exporting firms. Such a mechanism, which would be present in most open-economy macroeconomic models with sticky prices, has however failed to find strong support in previous empirical studies [see in particular Bodnar and Gentry (1993) and Friberg and Nydahl (1999)]. Furthermore, it should apply only to exporting firms, and not to entire stock market indices, which we use in our empirical application. Moreover, that alternative explanation could not account for the intertemporal increase in the correlation, nor for the positive link between equity market development and the strength of the correlation, which we present in section 7.4.

### **6.3 Exchange rate returns and portfolio flows**

Data on equity flows allow for another test of our model. Model implication 3 highlights a positive correlation between equity investment into the foreign market and the foreign currency return. Data on bilateral equity flows relative to the United States are unfortunately available only at the monthly frequency. Table 5 summarizes the evidence on the correlation of net U.S. flows into the same 17 OECD countries as before and the corresponding forex rate returns.

Only France and Portugal show positive correlation at the 1% significance level for the entire data period since 1987. Pooling the entire data for all countries even produces a negative, albeit insignificant, correlation. However, this picture is reversed for the more recent data period since 1990. The correlation is now significantly positive at the 1% level for six countries. It is positive but insignificant for four others. The correlation for the pooled sample increases to 0.114 and is statistically significant at the 1% level. Overall, the evidence is supportive of a linkage between net equity portfolio flows and exchange rate returns. Net equity flows into the foreign market are positively correlated with an appreciating foreign currency.

---

<sup>29</sup> Lane and Milesi-Ferretti (2003) also report a negative correlation between domestic real return in local currency and the real exchange rate at the yearly horizon.

<sup>30</sup> It would be desirable to allow for nontrivial interest rate determination in our model so as to analyze jointly the uncovered interest and equity parity conditions. We leave this complex extension for further research.

**Table 5**  
Correlation of exchange rate returns and net foreign equity inflows

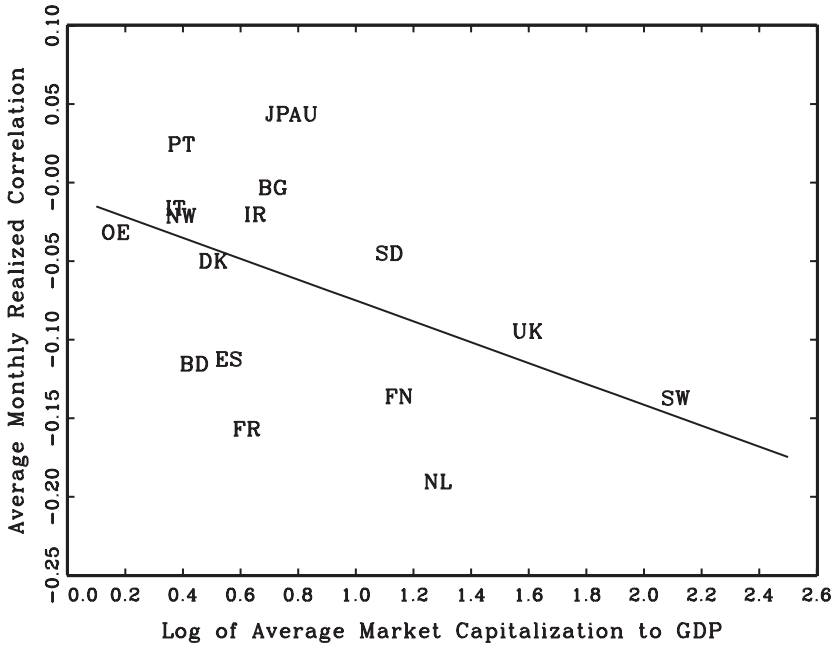
	$\text{corr}(-dE_t, dK_t^f - dK_t^{ht})$		
	(a) January 1980– December 2001	(b) January 1990– December 2001	(c) January 1995– December 2001
Australia	0.0112	-0.0478	-0.0010
Austria	-0.1155***	0.2051***	0.2740***
Belgium-Luxembourg	0.1208	0.2541***	0.3846***
Denmark	-0.0938**	-0.0174	-0.0295
Finland	-0.0002	-0.0194	0.0473
France	0.1400**	0.1539**	0.1814**
Germany	-0.0872	-0.0412	0.1210
Ireland	0.0445	0.1461	0.0769
Italy	-0.0071	0.0824	0.1936**
Japan	0.0382	0.0292	-0.0620
Netherlands	-0.0745	-0.0265	-0.0279
Norway	-0.0162	0.0033	-0.0125
Portugal	0.1844***	0.1971***	0.1582***
Spain	0.0586	0.1521***	0.1939***
Sweden	0.0235	0.0701	0.3620***
Switzerland	0.1061*	0.1608*	0.3052***
U.K.	0.0775	-0.0197	0.0716
Mean	0.0274	0.0754	0.1316
SD	0.0824	0.1004	0.1413
Pooled data	-0.0026	0.0665***	0.1145***

Reported are correlations of the exchange rate return,  $-dE_t$ , and net foreign stock ownership increase by U.S. residents,  $dK_t^f - dK_t^{ht}$ , for various sample periods. Net foreign stock ownership increase (or net foreign inflow) is defined as net U.S. purchases of foreign equities minus net foreign purchases of U.S. equities, and normalized as a proportion of the average absolute level of net foreign stock ownership increase by U.S. residents over the previous 12 months. The theory predicts  $\text{corr}(-dE_t, dK_t^f - dK_t^{ht}) > 0$ . We test whether the correlation is significantly different from zero using robust standard errors and denote by \*, \*\*, and \*\*\* significance at a 10, 5, and 1% level, respectively. The last row provides the result for the pooled data.

#### 6.4 Equity market development and the correlation structure

The evidence in Tables 2, 3, and 4 suggest that foreign equity excess returns became a more important determinant of exchange rate behavior in the 1990s, presumably because of increased equity market development and integration. We can test this hypothesis further by examining the cross-sectional variation of equity market development within the OECD sample. Two crude measures of equity market development are given by the quarterly market capitalization of the OECD country relative to its GDP as a proxy for internal capital market development and by the gross equity trade with the United States relative to GDP as a proxy for international market integration. These measures of equity market development are highly correlated at 0.84.

Figure 1 plots the average monthly correlation between local equity excess returns (over U.S. returns) and FX returns for the 17 countries as a function of the (log) market capitalization to GDP ratio for the sample period 1995–2001. Countries with higher equity market development tend to show a more negative correlation between their equity market excess



**Figure 1**  
 Plotted are the average monthly realized correlation of excess equity returns (defined as local index returns over U.S. index returns) and the corresponding foreign exchange return (in dollar terms) as a function of the log average market capitalization to GDP ratio for 17 OECD countries over the period 1995–2001. The countries are Australia (AU), Austria (OE), Belgium-Luxembourg (BG), Denmark (DK), Finland (FN), France (FR), Germany (BD), Ireland (IR), Italy (IT), Japan (JP), Netherlands (NL), Norway (NW), Portugal (PT), Spain (ES), Sweden (SD), Switzerland (SW), and the U.K. (UK).

return and the exchange rate return. We can analyze this link further using panel regressions reported in Table 6. We calculate quarterly realized correlations from daily returns for all 17 OECD countries and regress those alternatively on both measures of market development and a fixed time effect for each quarter. Both market development proxies are statistically significant at a 1% level. We conclude that the correlation structure of equity and exchange rate returns is related to the level of equity market development. The correlation is more negative for OECD countries with the most developed equity markets.

## 7. Conclusion

This article develops a new integrated analysis of exchange rates, equity prices, and equity portfolio flows. Such a framework is warranted by the increasing magnitude of international equity flows over the last decade. We argue that the integration of equity markets does not imply convergence to a financial structure based on full exchange rate risk trading. The

**Table 6**  
**Correlation structure and stock market development**

Specification	Coefficients ( $n = 724$ )		Adjusted $R^2$
	$\alpha$	$\beta$	
I	-0.0080 (0.0384)	-0.0715*** (0.0231)	0.292
II	0.1046*** (0.0403)	-0.0199*** (0.0051)	0.295

Reported are the panel regressions of the quarterly realized correlations ( $QRcorr_{it}$ ) between foreign stock market excess returns and exchange rate returns on two alternative measures of stock market development and fixed time effects  $D_t$  for each quarter of the sample period 1990–2002:

$$I: QRcorr_{it} = \alpha + \beta \log(Mcap_{it}/GDP_{it}) + \gamma D_t + \varepsilon_{it}$$

$$II: QRcorr_{it} = \alpha + \beta \log(Tvol_{it}/GDP_{it}) + \gamma D_t + \varepsilon_{it}$$

Quarterly-realized correlations are calculated based on daily equity market excess returns for 17 OECD countries ( $i = 1, 2, \dots, 17$ ) relative to the U.S. equity market return and daily exchange rate returns of the respective dollar exchange rate. Market development is alternatively measured by the ratio of quarterly capital market capitalization ( $Mcap_{it}$ ) to GDP or the ratio of quarterly cross-border equity trading volume with the U.S. ( $Tvol_{it}$ ) to GDP. We report in parenthesis robust standard errors (Newey–West) and allow for first-order serial autocorrelation of the error. Fixed effects are not reported.

available evidence from U.S. global mutual funds suggests to the contrary that forex risk in international equity portfolios is mostly unhedged and therefore not internationally traded. The main theoretical contribution of this article is to explore the implications of incomplete forex risk trading for the correlation structure of exchange rate and equity returns and exchange rate returns and net portfolio flows.

The theoretical analysis incorporates a stylized fact from the recent microstructure research on exchange rates, namely that net forex order flow tends to generate large and persistent exchange rate changes. But the forex order flow itself is tied to endogenous portfolio flows which emerge under optimal dynamic investment in an incomplete market setting. The entire exchange rate dynamics is therefore based exclusively on the financial market structure as opposed to traditional macroeconomic determinants.

We highlight three dimensions in which this parsimonious approach is successful. First, the model can explain a large degree of exchange rate volatility if the elasticity of forex liquidity supply is sufficiently low. Second, we derive a negative correlation between foreign equity excess returns (in local currency) and the corresponding exchange rate returns. This correlation contradicts the conventional belief that strong equity markets are accompanied by currency appreciation. It is induced by the rebalancing of the portfolio of global investors who decrease the exposure of their investments to exchange rate risk. This correlation structure has not been highlighted in the previous exchange rate literature. Such a negative correlation decreases the risk of foreign investment in home currency terms as negative foreign equity returns tend to be compensated

by positive exchange rate returns. This automatic hedge reduces the home bias and facilitates international equity risk sharing. We find very strong empirical support for the predicted return correlation at daily, monthly, and quarterly horizons. Stock return differentials explain as much as 30% of the variance of the exchange rate at the quarterly frequency for some countries. Moreover, the negative correlation becomes more pronounced after 1990, perhaps because of more developed and integrated international equity markets. The cross-sectional evidence also points to the role of financial market development. Countries with a higher equity market capitalization relative to GDP tend to have a more negative return correlation. Third, we explore the correlation between exchange rate returns and net equity flows. The model predicts a positive correlation since net equity flows are tied to forex order flows. The period after 1990 shows a statistically highly significant positive correlation for the pooled data of 17 OECD countries. Also, the economic significance of this correlation increases after 1995 to 11% in the pooled data.

Our analysis can be extended in both the theoretical and empirical dimensions. Theoretical work could account for asymmetric information or differences in opinion concerning the international equity returns between the home and foreign investors [Shiller, Kon-Ya, and Tsutsui (1996)]. This would introduce an additional and potentially important new motive for portfolio flows alongside the exchange rate risk rebalancing motive highlighted in the present framework. Another interesting extension consists in relaxing our assumption of perfectly elastic bond supplies and allow for dynamic interest rate differentials between the two countries. Since a home interest rate decrease is related to a home equity price increase simply through the discount rate and since a home equity increase is in turn related to a home currency depreciation, we expect our framework to produce the systematic violations of uncovered interest rate parity reported in the literature. Theoretical work could also generalize the utility specification to study the links between equity flows, consumption volatility, and exchange rate risk [Bekaert, Harvey, and Lundblad, (2004)].

Future empirical work should test the portfolio balancing channel at the aggregate level [Hau and Rey (2004) and Siourounis (2004)] or using investor or fund-specific data. This promising direction is pursued by Broner, Gelos, and Reinhart (2004). Microdata should ultimately allow us to evaluate the empirical importance of the portfolio rebalancing motive relative to other information or belief-based trading motives in explaining aggregate equity flows. On a more general level, we need a better understanding of the source of equity market incompleteness and why for example forex hedging seems prevalent for bond funds, but not for equity funds. Furthermore, we would like to know more about the role forex derivatives in investment strategies of investors and funds and how permission to use such derivatives changes investment allocations. Indeed the underlying reason for incomplete forex risk



trading certainly matters for policy conclusions. If forex risk hedging strategies are perceived to be too expensive in terms of transaction costs, then a more liquid market for long-run forex hedging instruments may allow for a shift towards a more efficient international risk sharing under which the documented correlation structure might disappears. If on the other hand agency problems are at the root of the problem, then improvement in market quality is unlikely to change the risk allocation.

## Appendix A: Equilibrium under Financial Autarchy

### Proposition 1:

Let  $F_t^h = f_0 + f_D D_t^h$  denote the fundamental equity value in the home country with  $f_D = 1/(\alpha_D + r)$  and  $f_0 = (r^{-1} - f_D)\bar{D}$ .

We conjecture a linear price equilibrium of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f. \end{aligned}$$

The excess payoffs over the riskless rate of home country equity follows as

$$dR_t^h = dP_t^h - rP_t^h dt + D_t^h dt = \alpha_\Psi^h \Psi_t^h dt + b_\Psi^h dw_t^h$$

with  $\Psi_t^h = (1, D_t^h)^T$  and coefficients  $\alpha_\Psi^h = (-rp_0, 1 - p_F)$  and  $b_\Psi^h = p_F f_D \sigma_D$ . The optimal asset demand for investors is given by

$$K_t^h = \frac{\mathcal{E}_t(dR_t^h)}{\rho \sigma_R^2 dt}$$

with  $\sigma_R^2 dt = \mathcal{E}_t(dR_t^h)^2$ . Market clearing requires  $K_t^h = 1$  and implies for the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho \sigma_R^2}{r} \\ p_F &= 1. \end{aligned}$$

The same price parameters are obtained for the foreign stock market. The instantaneous volatility of the excess payoff is given by

$$\sigma_R^2 = (b_\Psi^h)^2 = \frac{\sigma_D^2}{(\alpha_D + r)^2}.$$

## Appendix B: Equilibrium under Complete Risk-Sharing

### Proposition 2:

We conjecture a linear price system of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1. \end{aligned}$$

As before, we denote by  $F_t^h = f_0 + f_D D_t^h$  and  $F_t^f = f_0 + f_D D_t^f$  the fundamental values of the two risky assets. The home country investor faces excess payoffs  $R_t^h$  and  $R_t^f$  for home and foreign equity, respectively. The foreign country investor (denoted by  $*$ ) faces excess payoffs (in foreign currency)  $R_t^{f*}$  and  $R_t^{h*}$  for foreign and home country equity, respectively. Linear approximations allow us to write:

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t \bar{P} + dP_t^f - dE_t dP_t^f - r[P_t^f - \bar{P}(E_t - 1)]dt + [D_t^f - \bar{D}(E_t - 1)]dt \\ dR_t^{f*} &= dP_t^f - rP_t^f dt + D_t^f dt \\ dR_t^{h*} &\approx dE_t \bar{P} + dP_t^h + dE_t dP_t^h - r[P_t^h + \bar{P}(E_t - 1)]dt + [D_t^h + \bar{D}(E_t - 1)]dt. \end{aligned}$$

The constant exchange rate ( $dE_t = 0$ ) implies that payoffs in foreign currency terms are equal to home currency payoffs, hence  $dR_t^{h*} = dR_t^h$ , and  $dR_t^{f*} = dR_t^f$ . The excess payoffs take on the simple form

$$\begin{aligned} dR_t^j &= \alpha_\psi^j \Psi_t^j dt + b_\psi^j dw_t^j \\ dR_t^{j*} &= \alpha_\psi^{j*} \Psi_t^j dt + b_\psi^{j*} dw_t^j \end{aligned}$$

where  $j = h, f$  denotes the country index,  $\Psi_t^j = (1, D_t^j)^T$  the state variable and  $\alpha_\psi^j = \alpha_{\psi}^{j*} = (\alpha_0^j, \alpha_b^j)$ ,  $b_\psi^j = b_{\psi}^{j*}$  coefficients.

Finally, we consider the correlation structure of the payoffs. Let  $\Omega dt$  denote the covariance matrix of the excess payoffs ( $dR_t^h, dR_t^f$ ) (in home currency terms) for the home investor and  $\Omega^* dt$  the corresponding covariance matrix of the excess payoffs ( $dR_t^{f*}, dR_t^{h*}$ ) (in foreign currency) for the foreign investor. Symmetry of the two-country model implies

$$\Omega = \Omega^* = \begin{pmatrix} \Omega_{11} & \Omega_{21} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}, \quad \Omega^{-1} = (\Omega^*)^{-1} = \frac{1}{\det \Omega} \begin{pmatrix} \Omega_{22} & -\Omega_{21} \\ -\Omega_{21} & \Omega_{11} \end{pmatrix}$$

with  $\det \Omega = \Omega_{11} \Omega_{22} - \Omega_{21} \Omega_{21}$ .

For the special case of complete markets with a constant exchange rate, we have  $\mathcal{E}_t(dE_t dP_t^h) = 0$ ,  $\Omega_{21} = 0$ , and  $\Omega_{11} = \Omega_{22} = \sigma_R^2$ . Therefore,

$$\Omega^{-1} = \frac{1}{\sigma_R^4} \begin{pmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{pmatrix} = \frac{1}{\sigma_R^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sigma_R^2} \mathbf{1}_{2 \times 2}.$$

The first-order condition for the asset demands is given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho dt} \mathcal{E}_t \begin{pmatrix} dR_t^h & dR_t^f \\ dR_t^{f*} & dR_t^{h*} \end{pmatrix} \Omega^{-1}. \tag{6}$$

Market clearing in the two stock markets ( $K_t^h + K_t^{h*} = 1$ ,  $K_t^f + K_t^{f*} = 1$ ) implies the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho \sigma_R^2}{2r} \\ p_F &= 1. \end{aligned}$$

For the optimal portfolio positions we obtain

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Hence, the existence and uniqueness of the complete risk-sharing equilibrium (proposition 2).

## Appendix C: Equilibrium under Incomplete Risk-Sharing

We prove the existence of the equilibrium under incomplete risk-sharing and its uniqueness in the class of linear equilibria. We proceed in four steps: (i) using our guessed solutions for exchange rates and equity prices, we derive optimal asset demands and the differential system governing the dynamics of our model (Appendix C1, proposition 3 in the article); (ii) we impose market clearing and identify the parameters (Appendix C2); (iii) we show the existence and uniqueness of  $z$  (Appendix C3); (iv) we show the existence and uniqueness of all the other parameters, thereby establishing the existence and uniqueness of the equilibrium (Appendix C4, proposition 4 in the article). Appendices C3 and C4 are printed only in the working paper version (NBER WP 9398).

### C1 Exchange rate dynamics

#### Proposition 3:

We assume that the exchange rate process is of the form  $E_t = 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t$  and that equity prices have the following representation

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h + p_\Delta \Delta_t + p_\Lambda \Lambda_t \\ P_t^f &= p_0 + p_F F_t^f - p_\Delta \Delta_t - p_\Lambda \Lambda_t. \end{aligned}$$

Let  $j = h, f$  denote the country index,  $\Psi_t^j = (1, D_t^j, \Delta_t, \Lambda_t)^T$  the state variable,  $\mathbf{dw}^j = (dw_t^j, dw_t)^T = (dw_t^j, dw_t^h - dw_t^f)^T$  a  $(1 \times 2)$  vector of innovations. For coefficients  $\alpha_\Psi^j = (\alpha_\Delta^j, \alpha_B^j, \alpha_\Delta^j, \alpha_\Lambda^j)$ ,  $\alpha_\Psi^{j*} = (\alpha_\Delta^{j*}, \alpha_B^{j*}, \alpha_\Delta^{j*}, \alpha_\Lambda^{j*})$ ,  $\mathbf{b}_\Psi^j = (p_F f_D \sigma_D, b_\Psi^j)$ ,  $\mathbf{b}_\Psi^{j*} = (p_F f_D \sigma_D, b_\Psi^{j*})$  we express excess payoffs as

$$\begin{aligned} dR_t^j &= \alpha_\Psi^j \Psi_t^j dt + \mathbf{b}_\Psi^j \mathbf{dw}_t^j \\ dR_t^{j*} &= \alpha_\Psi^{j*} \Psi_t^j dt + \mathbf{b}_\Psi^{j*} \mathbf{dw}_t^j \end{aligned}$$

and the first-order conditions imply for the optimal asset demand

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho} \begin{pmatrix} \alpha_\Psi^h \Psi_t^h & \alpha_\Psi^f \Psi_t^f \\ \alpha_\Psi^{f*} \Psi_t^f & \alpha_\Psi^{h*} \Psi_t^h \end{pmatrix} \Omega^{-1}.$$

Market clearing  $(K_t^h + K_t^{h*} = 1, K_t^{f*} + K_t^f = 1)$  gives

$$\begin{aligned} K_t^{h*} - K_t^f &= \frac{1}{\rho} [m_\Delta \Delta_t + m_\Lambda \Lambda_t] \\ dK_t^{h*} - dK_t^f &= \frac{1}{\rho} [-\alpha_D m_\Delta \Delta_t dt + z m_\Lambda \Lambda_t dt] + \frac{1}{\rho} [m_\Delta \sigma_D + m_\Lambda \sigma_\Lambda] dw_t \end{aligned}$$

where we define the coefficients

$$\begin{aligned}
 m_\Delta &= 2p_\Delta(\alpha_D + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2m_\alpha e_\Delta \Omega_{22}^{-1} \\
 m_\Lambda &= 2p_\Lambda(-z + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2m_z e_\Lambda \Omega_{22}^{-1} \\
 m_\alpha &= (\alpha_D + r)\bar{P} - \bar{D} \\
 m_z &= (-z + r)\bar{P} - \bar{D}
 \end{aligned}$$

Finally, we replace

$$\Lambda_t = \frac{1}{e_\Lambda}(E_t - \bar{E}) - \frac{e_\Delta}{e_\Lambda}\Delta_t$$

and find that the term  $(K_t^{h^*} - K_t^f)\bar{D}dt + (dK_t^f - dK_t^{h^*})\bar{P}$  is linear in  $E_t - \bar{E}, \Delta_t$  and  $dw_t$ . Substitution in the forex order flow constraint (4) implies that the exchange rate process can be represented as:

$$dE_t = k_1\Delta_t + k_2(E_t - \bar{E}) + k_3dw_t.$$

The whole model is therefore amenable to Equation (5) of the article (proposition 3).

### C2 Identification of the parameters

Market clearing in the two stock markets ( $K_t^h + K_t^{h^*} = 1, K_t^f + K_t^{f^*} = 1$ ) implies four parameter constraints (one for each element in  $\Psi_t^j = (1, D^j, \Delta_t, \Lambda_t)^T$ ) given by

$$p_0 = \frac{-\rho \det\Omega - (\Omega_{11} - \Omega_{12})\mathcal{E}_t(dE_t dP_t^f)/dt}{r(\Omega_{11} - 2\Omega_{12} + \Omega_{22})} \tag{7}$$

$$p_f = 1 \tag{8}$$

$$p_\Delta = -e_\Delta \frac{m_\alpha(\Omega_{21} + \Omega_{11})}{(\alpha_D + r)\bar{\Omega}} \tag{9}$$

$$p_\Lambda = -e_\Lambda \frac{m_z(\Omega_{21} + \Omega_{11})}{(-z + r)\bar{\Omega}} \tag{10}$$

where we define  $\bar{\Omega} = \Omega_{11} + 2\Omega_{21} + \Omega_{22}$ . The forex order flow constraint (4) implies an additional three constraints (for  $\Delta_t, \Lambda_t, dw_t$ ) given by

$$e_\Delta(\bar{K}\bar{D} - \kappa\alpha_D) + m_\Delta \frac{1}{\rho}(\bar{D} + \alpha_D\bar{P}) = -\bar{K} \tag{11}$$

$$e_\Lambda(\bar{K}\bar{D} + \kappa z) + m_\Lambda \frac{1}{\rho}(\bar{D} - z\bar{P}) = 0 \tag{12}$$

$$e_\Delta\kappa\sigma_D + e_\Lambda\kappa - m_\Delta \frac{1}{\rho}\bar{P}\sigma_D - m_\Lambda \frac{1}{\rho}\bar{P} = 0. \tag{13}$$

These seven equations determine the seven parameters  $p_0, p_f, p_\Delta, p_\Lambda, e_\Delta, e_\Lambda$ , and  $z$  as a function of  $\bar{P}, \bar{\Lambda}$  and  $\bar{K}$  as well as the parameters of the dividend process  $(\alpha_D, \bar{D}, \sigma_D)$ , the

elasticity of the forex liquidity supply,  $\kappa$ , and the investor risk aversion  $\rho$ . Moreover, steady-state levels  $\bar{P} > 0$ ,  $\bar{\Lambda}$  and  $0 < \bar{K} < 1$  are equal to

$$\begin{aligned}\bar{P} &= p_0 + \frac{\bar{D}}{r} + p_{\Lambda}\bar{\Lambda} = p_0 + \frac{\bar{D}}{r} \\ \bar{K} &= \frac{\rho[\Omega_{11} - \Omega_{21}] - \mathcal{E}_t(dE_t dP_t^f)/dt}{\rho(\Omega_{11} - 2\Omega_{21} + \Omega_{22})} \\ \bar{\Lambda} &= 0.\end{aligned}$$

The covariances are given by

$$\begin{aligned}\Omega_{11} &= (f_D\sigma_D)^2 + 2[p_{\Delta}\sigma_D + p_{\Lambda}]^2 + 2f_D\sigma_D[p_{\Delta}\sigma_D + p_{\Lambda}] \\ \Omega_{12} &= -2(p_{\Delta}\sigma_D + p_{\Lambda})^2 - [2(p_{\Delta}\sigma_D + p_{\Lambda}) + f_D\sigma_D]\bar{P}(e_{\Delta}\sigma_D + e_{\Lambda}) - 2(p_{\Delta}\sigma_D + p_{\Lambda})f_D\sigma_D \\ \Omega_{22} &= (f_D\sigma_D)^2 + 2[\bar{P}(e_{\Delta}\sigma_D + e_{\Lambda}) + p_{\Delta}\sigma_D + p_{\Lambda}]^2 + 2f_D\sigma_D[\bar{P}(e_{\Delta}\sigma_D + e_{\Lambda}) + p_{\Delta}\sigma_D + p_{\Lambda}]\end{aligned}$$

and furthermore

$$\bar{\Omega} = 2(f_D\sigma_D)^2 + 2[\bar{P}(e_{\Delta}\sigma_D + e_{\Lambda})]^2, \quad (14)$$

where  $\bar{\Omega}(z) = \Omega_{11} + 2\Omega_{21} + \Omega_{22} > 0$  represents the instantaneous excess payoff variance of the total market portfolio of all domestic and foreign equity.

## Appendix D: Infinitely Elastic Supply and Complete Risk Sharing

### Proposition 5:

For a completely price elastic currency supply ( $\kappa \rightarrow \infty$ ), the exchange rate is constant at  $\bar{E} = 1$ . This requires that  $e_{\Delta} = e_{\Lambda} = 0$ . It follows that  $c_{\Delta} = c_{\Lambda} = 0$  and  $\mathbf{Cp} = \mathbf{d}$  (see Appendix C4 of the working paper version) implies  $p_{\Delta} = p_{\Lambda} = 0$ . The latter gives  $m_{\Delta} = m_{\Lambda} = 0$ . Moreover, since  $\mathcal{E}_t(dE_t dP_t^f) = 0$  and  $\Omega_{21} = 0$ , we obtain steady-state equity holdings

$$\bar{K} = \frac{\Omega_{11}}{(\Omega_{11} + \Omega_{22})} = \frac{1}{2},$$

which correspond to full equity risk sharing as in proposition 2.

## Appendix E: Correlation Structure

### Corollary 1:

The symmetry of the model implies  $\mathcal{E}_t(dE_t dR_t^h) = -\mathcal{E}_t(dE_t dR_t^f)$ . Furthermore,

$$\mathcal{E}_t(dE_t dR_t^h)/dt = (e_{\Delta}\sigma_D + e_{\Lambda})[f_D\sigma_D + 2(p_{\Delta}\sigma_D + p_{\Lambda})] < 0$$

amounts to showing that  $\bar{e} \equiv e_{\Delta}\sigma_D + e_{\Lambda} < 0$  and  $f_D\sigma_D + 2(p_{\Delta}\sigma_D + p_{\Lambda}) > 0$ . We note that the latter follows for some  $\kappa > \bar{\kappa}$  since  $p_{\Delta}\sigma_D + p_{\Lambda} \rightarrow 0$  for  $\kappa \rightarrow \infty$  (see proposition 5) and  $f_D\sigma_D > 0$ . To simplify notations we define

$$m = \frac{(\bar{K}D - \alpha_D\kappa)\bar{P}}{(\bar{D} + \alpha_D\bar{P})}, \quad n = \frac{(\bar{K}D + z\kappa)\bar{P}}{(\bar{D} - z\bar{P})}.$$

Clearly,  $m < 0$  and  $n < 0$  under the parameter constraints of proposition 4. Moreover,  $m - n < 0$ , because (for  $\alpha_D > -z$ ) we find

$$(\bar{D} - z\bar{P})(\overline{KD} - \alpha_D\kappa) - (\bar{D} + \alpha_D\bar{P})(\overline{KD} + z\kappa) = -(\alpha_D + z)[\bar{D}\kappa + \overline{PKD}] < 0.$$

Substituting Equations (11) and (12) into (13) implies

$$e_{\Delta}\sigma_D[\kappa + m] + e_{\Lambda}[\kappa + n] = \frac{-\overline{KP}\sigma_D}{(\bar{D} + \alpha_D\bar{P})} < 0.$$

Subtracting the term  $e_{\Delta}\sigma_D(m - n) > 0$  (because  $e_{\Delta} < 0$ ) from the left hand side implies  $e_{\Delta}\sigma_D[\kappa + n] + e_{\Lambda}[\kappa + n] < 0$ . Therefore

$$e_{\Delta}\sigma_D + e_{\Lambda} < 0,$$

since  $\kappa + n > 0$  is trivially fulfilled (for  $\kappa > \bar{\kappa}$ ,  $\bar{K} > 0$ ,  $\bar{D} > 0$ ,  $\bar{P} > 0$ ). Hence  $\mathcal{E}(dE_t dR_t^h) < 0$ .

**Corollary 2:**

The home and foreign excess payoff processes (in local currency) and the exchange rate returns are

$$\begin{aligned} dR_t^h &= \alpha_0^h dt + \alpha_D^h D_t^h dt + \alpha_{\Delta}^h \Delta_t dt + \alpha_{\Lambda}^h \Lambda_t dt + p_{FD} f_D \sigma_D dw_t^h + (p_{\Delta} \sigma_D + p_{\Lambda}) dw_t \\ dR_t^{f*} &= \alpha_0^{f*} dt + \alpha_D^{f*} D_t^{f*} dt + \alpha_{\Delta}^{f*} \Delta_t dt + \alpha_{\Lambda}^{f*} \Lambda_t dt + p_{FD} f_D \sigma_D dw_t^f - (p_{\Delta} \sigma_D + p_{\Lambda}) dw_t \\ dE_t &= -e_{\Delta} \alpha_D \Delta_t dt - e_{\Lambda} \alpha_D \Lambda_t dt + (e_{\Delta} \sigma_D + e_{\Lambda}) dw_t \end{aligned}$$

and the relative payoff follows as

$$dR_t^{f*} - dR_t^h = 2p_{\Delta}(\alpha_D + r)\Delta_t dt + 2p_{\Lambda}(-z + r)\Lambda_t dt - [f_D \sigma_D + 2p_{\Delta} \sigma_D + 2p_{\Lambda}] dw_t.$$

Hence, we obtain a perfect negative return correlation,

$$\text{corr} \left[ -dE_t, (dR_t^{f*} - dR_t^h) / \bar{P} \right] = -1 < 0.$$

**Corollary 3:**

The net foreign equity inflows are given by

$$\begin{aligned} dK_t^f - dK_t^{h*} &= \frac{1}{\rho} [\alpha_D m_{\Delta} \Delta_t dt - z m_{\Lambda} \Lambda_t dt] - \frac{1}{\rho} [m_{\Delta} \sigma_D + m_{\Lambda} \sigma_{\Lambda}] dw_t \\ \mathcal{E}_t \left( dE_t, dK_t^f - dK_t^{h*} \right) &= -\frac{2\kappa}{\bar{P}} (e_{\Delta} \sigma_D + e_{\Lambda})^2 dt \end{aligned}$$

Hence

$$\text{corr} \left[ -dE_t, \left( dK_t^f - dK_t^{h*} \right) \right] = 1 > 0.$$

**References**

Bekaert, G., and C. Harvey, 2000, "Foreign Speculators and Emerging Equity Markets," *Journal of Finance*, 55(2), 565–613.  
 Bekaert, G., C. Harvey, and R. L. Lumsdaine, 2002, "The Dynamics of Emerging Market Equity Flows," *Journal of International Money and Finance*, 21(3), 295–350.  
 Bekaert, G., C. Harvey, and C. Lundblad, 2004, "Growth Volatility and Financial Liberalization," Working Paper 10560, NBER.

- Bodnar, G., and W. Gentry, 1993, "Exchange Rate Exposure and Industry Characteristics: Evidence from Canada, Japan and the USA," *Journal of International Money and Finance*, 44, 1755–1785.
- Bjønnes, G. H., D. Rime, and H. O. A. Solheim, 2005, "Liquidity Provision in the Overnight Foreign Exchange Market," *Journal of International Money and Finance*, 24, 175–196.
- Board of Governors of the Federal Reserve System, 2004, "Report on U.S. Holdings of Foreign Long-Term Securities," Washington, DC.
- Branson, W., and W. Henderson, 1985, "The Specification and Influence of Asset Markets," in R. Jones and P. Kenen (eds), *Handbook of International Economics*, North-Holland, Amsterdam.
- Brennan, M. J., and H. H. Cao, 1997, "International Portfolio Investment Flows," *Journal of Finance*, 52, 1851–1880.
- Broner, F., G. R. Gelos, and C. Reinhart, 2004, "When in Peril, Retrench: Testing the Portfolio Channel of Contagion," Working Paper 04/131, IMF.
- Brooks, R., H. Edison, M. Kumar, and T. Slok, 2001, "Exchange Rates and Capital Flows," Working Paper 01/190, IMF.
- Carlson, J., and C. Osler, 2000, "Rational Speculators and Exchange Rate Volatility," *European Economic Review*, 44, 231–253.
- Cashin, P., L. Cespedes, and R. Sahay, 2002, "Keynes, Cocoa and Copper: In Search of Commodity Currencies," Working Paper 02/223, IMF.
- Chen, Y., and K. Rogoff, 2002, "Commodity Currencies and Empirical Exchange Rate Equations," Working Paper 02/27, IMF.
- Evans, M., and R. Lyons, 2002a, "Order Flow and Exchange Rate Dynamics," *Journal of Political Economy*, 110(1), 170–180.
- Evans, M., and R. Lyons, 2002b, "Time-Varying Liquidity in Foreign Exchange," *Journal of Monetary Economics*, 49(5), 1025–1051.
- Evans, M., and R. Lyons, 2002c, "Informational Integration and FX Trading," *Journal of International Money and Finance*, 21(6), 807–831.
- Evans, M., and R. Lyons, 2003, "How Is Macro News Transmitted to Exchange Rates?" Working Paper 9433, NBER.
- Frankel, J., 1994, *The Internationalization of Equity Markets*, University of Chicago Press, Chicago, 1–20.
- Frankel, J., and A. Rose, 1995, "Empirical Research on Nominal Exchange Rates," in G. Grossman and K. Rogoff (eds), *Handbook of International Economics*, North-Holland, Amsterdam.
- Friberg, R., and S. Nydahl, 1999, "Openness and the Exchange Rate Exposure of the National Stock Markets," *International Journal of Finance and Economics*, 4, 55–62.
- Froot, K. A., and T. Ramadorai, 2005, "Currency Returns, Intrinsic Value, and Institutional Investor Flows," *Journal of Finance*, 60, 1535–1566.
- Froot, K., P. O'Connell, and M. Seasholes, 2001, "The Portfolio Flows of International Investors," *Journal of Financial Economics*, 59, 151–193.
- Griffin, J., F. Nardari, and R. Stulz, 2002, "Daily Cross-Border Equity Flows: Pushed or Pulled?" Working Paper 9000, NBER.
- Hau, H., 1998, "Competitive Entry and Endogenous Risk in the Foreign Exchange Market," *Review of Financial Studies*, 11(4), 757–787.
- Hau, H., and H. Rey, 2003, "Exchange Rates, Equity Prices and Capital Flows," Working Paper 9398, NBER.

- Hau, H., and H. Rey, 2004, "Can Portfolio Rebalancing Explain the Dynamics of Equity Returns, Equity Flows, and Exchange Rates?" *American Economic Review*, 96(2), 126–133.
- Hau, H., W. Killeen, and M. Moore, 2002. "How Has the Euro Changed the Foreign Exchange Market?" *Economic Policy*, 34, 151–177.
- Henry, P. B., 2000, "Do Stock Market Liberalizations Cause Investment Booms?" *Journal of Financial Economics*, 58(1–2), 301–334.
- Karolyi, G. A., and R. M. Stulz, 2002, "Are Financial Assets Priced Locally or Globally?" Working Paper 8994, NBER.
- Killeen, W., R. Lyons, and M. Moore, 2006, "Fixed versus Floating: Lessons from the EMS Order Flow," forthcoming in *Journal of International Money and Finance*.
- Lane, P., 2001, "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, 54(2), 235–66.
- Lane, P., and G. M. Milesi-Ferretti, 2003, "International Financial Integration," Working Papers 03/86, IMF.
- Levich, R. M., G. S. Hayt, and B. A. Ripston, 1999, "1998 Survey of Derivative and Risk Management Practices by U.S. Institutional Investors," Survey conducted by the NYU Salomon Center, CIBC World Markets, and KPMG, available at <http://www.stern.nyu.edu/rlevich/>.
- Lucas, R., 1982, "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, 10(3), 335–359.
- Lyons, R., 2001, *The Microstructure Approach to Exchange Rates*, MIT Press, Cambridge, MA.
- Meese, R., and K. Rogoff, 1982, "The Out-of-Sample Failure of Empirical Exchange Rate Models: Sampling Error or Misspecification?" International Finance Discussion Paper 204, Board of Governors of the Federal Reserve System.
- Meese, R., and K. Rogoff, 1983, "Empirical Exchange Rate Models of the Seventies: Do they Fit Out-of-Sample?" *Journal of International Economics*, 14(2), 3–24.
- Obstfeld, M., and K. Rogoff, 1995, "Exchange rate Dynamics Redux," *Journal of Political Economy*, 103, 624–60.
- Obstfeld, M., and K. Rogoff, 2000, "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, 50, 117–153.
- Obstfeld, M., and A. Taylor, 2002, "Globalization and Capital Markets," Working Paper 8846, NBER.
- Osler, C., 1998, "Short-Term Speculators and the Puzzling Behavior of Exchange Rates," *Journal of International Economics*, 43(1), 37–58.
- Perold, A. F., and E. C. Schulman, 1988, "The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards," *Financial Analyst Journal*, 44, 45–52.
- Portes, R., and H. Rey, 2005, "The Determinants of Cross-Border Equity Flows," *Journal of International Economics*, 65, 269–296.
- Reinhart, C. M., and K. Rogoff, 2004, "The Modern History of Exchange Rate Arrangements: A Reinterpretation," *Quarterly Journal of Economics*, 119(1), 1–48.
- Richards, A., 2005, "Big Fish in Small Ponds: The Trading Behavior and Price Impact of Foreign Investors in Asian Emerging Equity Markets," *Journal of Financial and Quantitative Analysis*, 40, 1–27.
- Rime, D., 2001, "Trading in Foreign Exchange Markets," PhD Thesis, Norwegian School of Management.
- Shiller, R. J., F. Kon-Ya, and Y. Tsutsui, 1996, "Why Did the Nikkei Crash? Expanding the Scope of Expectations Data Collection," *Review of Economics and Statistics*, 78(1), 156–64.



Shleifer, A., and R. W. Vishny, 1997, "The Limits of Arbitrage," *Journal of Finance*, 52(1), 35–55.

Siourounis, G., 2004, "Capital Flows and Exchange Rates: an Empirical Analysis," mimeo, London Business School.

Stulz, R., 1999, "Globalization of Equity Markets and the Cost of Capital," Working Papers 7021, NBER.

Warnock, F., B. Grier, and G. Lee, 2001, "The U.S. System for Measuring Cross-Border Investment in Securities: A Primer with a Discussion of Recent Developments," *Federal Reserve Bulletin*, 87(10), 633–650.